

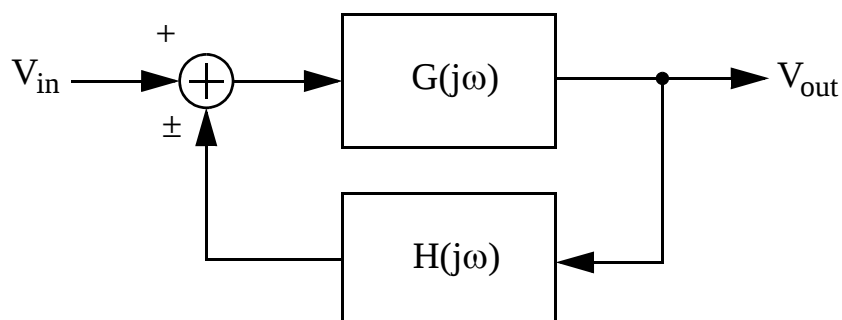
# CHAPTER 9

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## *OSCILLATORS*

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## THE PRINCIPLE OF POSITIVE OR NEGATIVE FEEDBACK



**Fig 9-1:** *The oscillator presented as a system of positive or negative feedback.*

An oscillator can be considered to be an amplifier with a transfer function  $G(j\omega)$ , combined with a feedback network with a transfer function  $H(j\omega)$  (cf Fig. 9.1). The system has the potential to oscillate at the frequencies for which the signal coming from the feedback block adds in phase with the input signal. This implies that the phase offset of the loop must be  $180^\circ$  (or rather  $180^\circ + k \cdot 360^\circ$ ) for negative feedback (- sign in Fig. 9-1) or equal to a multiple of  $360^\circ$  for positive feedback (+ sign in Fig. 9-1). In order for oscillations to establish themselves, it is also necessary that the signal be amplified each time it passes through the feedback loop. This implies that the gain of the loop must be greater than one. Under these conditions, the amplitude of the oscillations will grow even if the input signal is reduced to zero. When the amplitude of the oscillations becomes large, the amplifier will start to saturate, which reduces the gain at the oscillation frequency. The oscillations will then become stable when the gain of the loop at the oscillation frequency becomes exactly equal to one.

## OSCILLATION CONDITIONS AND THE BARKHAUSEN CRITERION

The output voltage of the system in Fig. 9-1 can be expressed as:

$$V_{out} = \frac{G(j\omega)}{1 \pm G(j\omega)H(j\omega)} V_{in} \quad (9.1)$$

with the + sign if there is negative feedback and the - sign if there is positive feedback.

For the oscillator, the output voltage is non-zero no matter what the input signal, even when this signal is zero. This last case is only possible when the gain  $G(j\omega)$  becomes infinite (which is not attainable) or when the denominator of (9.1) becomes zero for a certain frequency in radians  $\omega_0$ :

$$G(j\omega_0)H(j\omega_0) = \mp 1 \quad (9.2)$$

Eqn. 9.2 implies one condition for the gain and one for the phase:

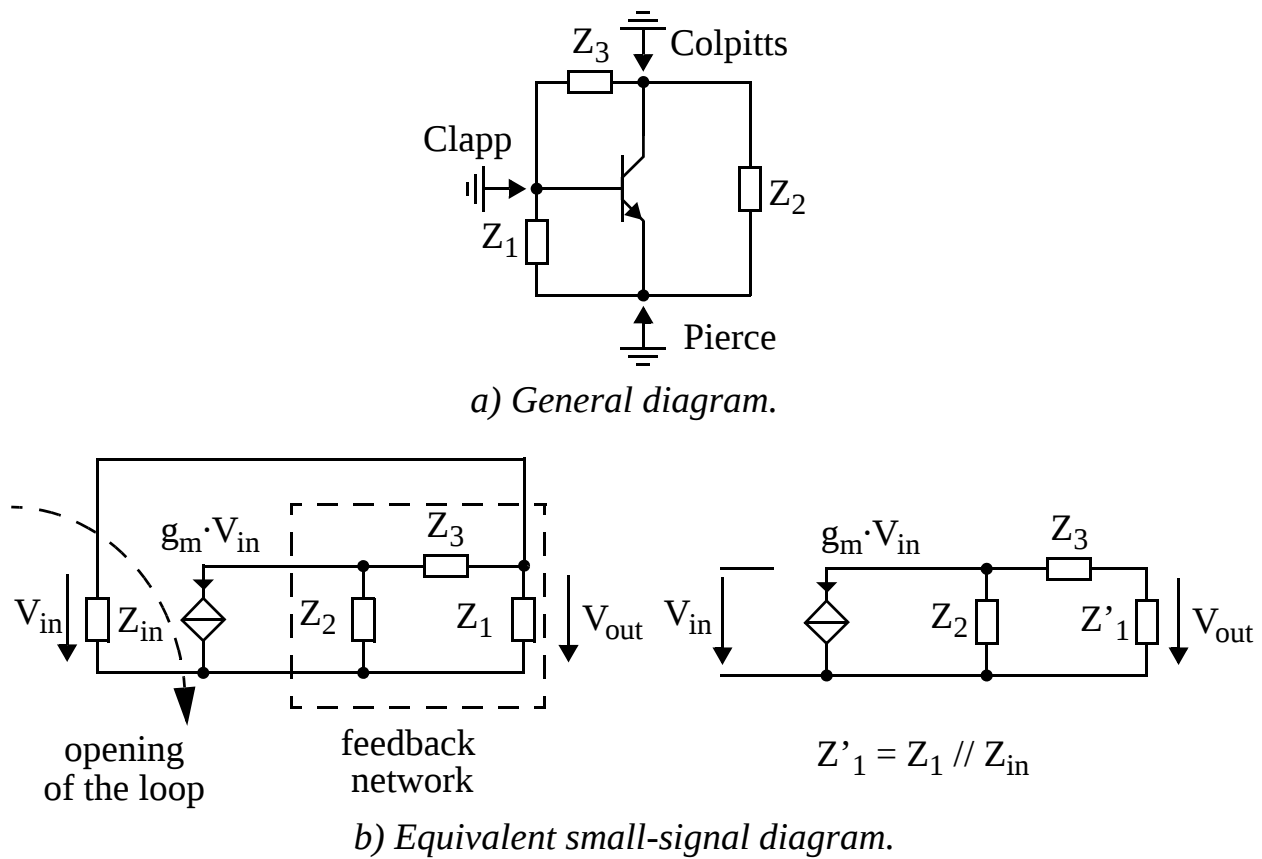
$$|G(j\omega_0)H(j\omega_0)| = 1 \quad (9.3)$$

$$\text{and: } \arg \{ G(j\omega_0)H(j\omega_0) \} = \begin{cases} 180^\circ & \text{for negative FB} \\ 0^\circ & \text{for positive FB} \end{cases} \quad (9.4)$$

In order for the system to oscillate, it is thus necessary that the gain of the loop have a modulus of exactly one with a total phase difference of the loop of  $180^\circ$  (negative FB) or zero (positive FB). The condition given by (9.3) corresponds to the *Barkhausen criterion*.

The feedback network usually determines the oscillation frequency. It often consists of a passive circuit with a bandpass transfer function, with no phase offset at the resonant frequency. For a wide-band circuit, the phase varies slightly around the resonant frequency and the harmonics created by the non-linearity of the amplifier are slightly attenuated. The more selective the circuit, the more the phase offset varies rapidly around the resonant frequency, and the output signal approaches a purely sinusoidal signal.

## ANALYSIS OF THE “THREE-POINT” OSCILLATOR (1/2)



**Fig 9-2:** “Three-point” oscillator.

The gain of the loop is given by:

$$-GH \equiv \frac{V_{out}}{V_{in}} = \frac{-g_m Z'_1 Z_2}{Z'_1 + Z_2 + Z_3} = \frac{-g_m}{Y'_1 (1 + Y_2 Z_3) + Y_2} \quad (9.5)$$

where the input impedance of the bipolar transistor  $Z_{in}$  has been included in  $Z'_1$  (and we can do the same with the output conductance of the bipolar transistor  $g_{ce}$  and  $Z_2$ , as well as the capacitance  $C_{bc}$  and  $Z_3$ ). The gain of the loop being negative (negative feedback), the oscillation condition requires it to be equal to  $-1$ , from which we get:

$$\boxed{g_m Z'_1 Z_2 + Z'_1 + Z_2 + Z_3 = 0} \quad (9.6)$$

or:

$$g_m + Y'_1 (1 + Y_2 Z_3) + Y_2 = 0 \quad (9.7)$$

## ANALYSIS OF THE “THREE-POINT” OSCILLATOR (2/2)

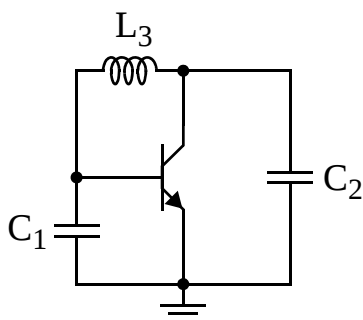
If the feedback network is composed only of reactive elements  $Z_i = j \cdot X_i$  for  $i = 1, 2, 3$  and the input impedance of the active element is purely resistive and equal to  $Z_{in} = r_\pi \equiv \beta/g_m$ , the oscillation condition then reduces to:

$$X_1[(1 + \beta)X_2 + X_3] - j \cdot r_\pi \cdot (X_1 + X_2 + X_3) = 0 \quad (9.8)$$

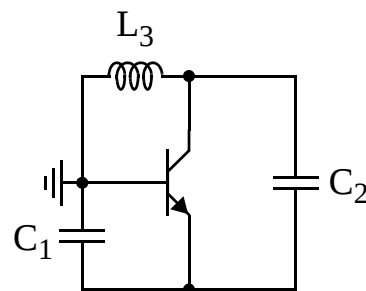
from which we get:

$$X_1 = \beta X_2 \quad (9.9)$$

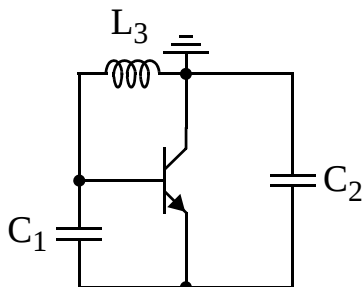
Since  $\beta > 0$ , Eqn. 9.9 implies that  $Z_1$  and  $Z_2$  must be reactances of the same type (with the same sign), either both capacitances or both inductances. Colpitts, Pierce and Clapp oscillators (cf Fig. 9-3 a, b, c) correspond to the case in which the impedances  $Z_1$  and  $Z_2$  are capacitances and  $Z_3$  is an inductance. The case in which  $Z_1$  and  $Z_2$  are inductances and  $Z_3$  is a capacitance corresponds to the Hartley oscillator (cf Fig. 9-3 d).



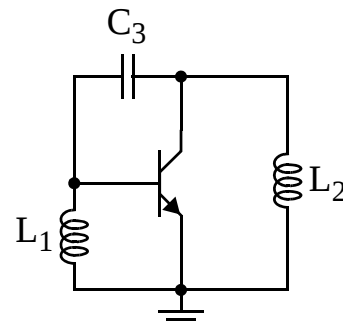
a) Pierce.



b) Clapp.



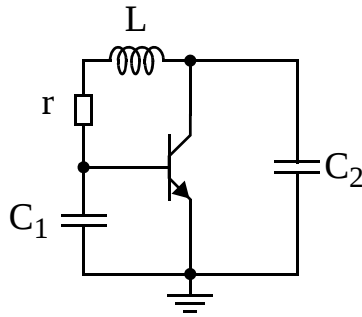
c) Colpitts.



d) Hartley.

**Fig 9-3:** Different types of oscillators.

## ANALYSIS OF THE PIERCE OSCILLATOR (1/2)



$$Y'_1 = \frac{g_m}{\beta} + j\omega C_1 \cong j\omega C_1$$

$$Y_2 = j\omega C_2$$

$$Z_3 = L = r + j\omega L$$

$r$  represents the losses in the inductor

**Fig 9-4:** *Pierce oscillator.*

By considering the input impedance of the transistor of the Pierce oscillator presented in Fig. 9-4 as being much less than the impedance of the capacitor  $C_1$  at the oscillation frequency, the oscillation condition (9.7) becomes:

$$g_m - \omega^2 r C_1 C_2 + j\omega(C_1 + C_2 - \omega^2 L C_1 C_2) = 0 \quad (9.10)$$

or:

$$\begin{cases} g_m - \omega^2 r C_1 C_2 = 0 \\ C_1 + C_2 - \omega^2 L C_1 C_2 = 0 \end{cases} \quad (9.11)$$

from which we get the oscillation frequency in radians  $\omega_0$ :

$$\omega_0 = \frac{1}{\sqrt{L C_{12}}} \quad \text{with:} \quad C_{12} \equiv \frac{C_1 C_2}{C_1 + C_2} \quad (9.12)$$

The losses of the inductances are compensated by the active element. The oscillations are maintained by a particular value  $g_{mcrit}$  of the transconductance given by:

$$g_{mcrit} \equiv \omega_0^2 r C_1 C_2 = \frac{\omega_0(C_1 + C_2)}{Q_L} \quad (9.13)$$

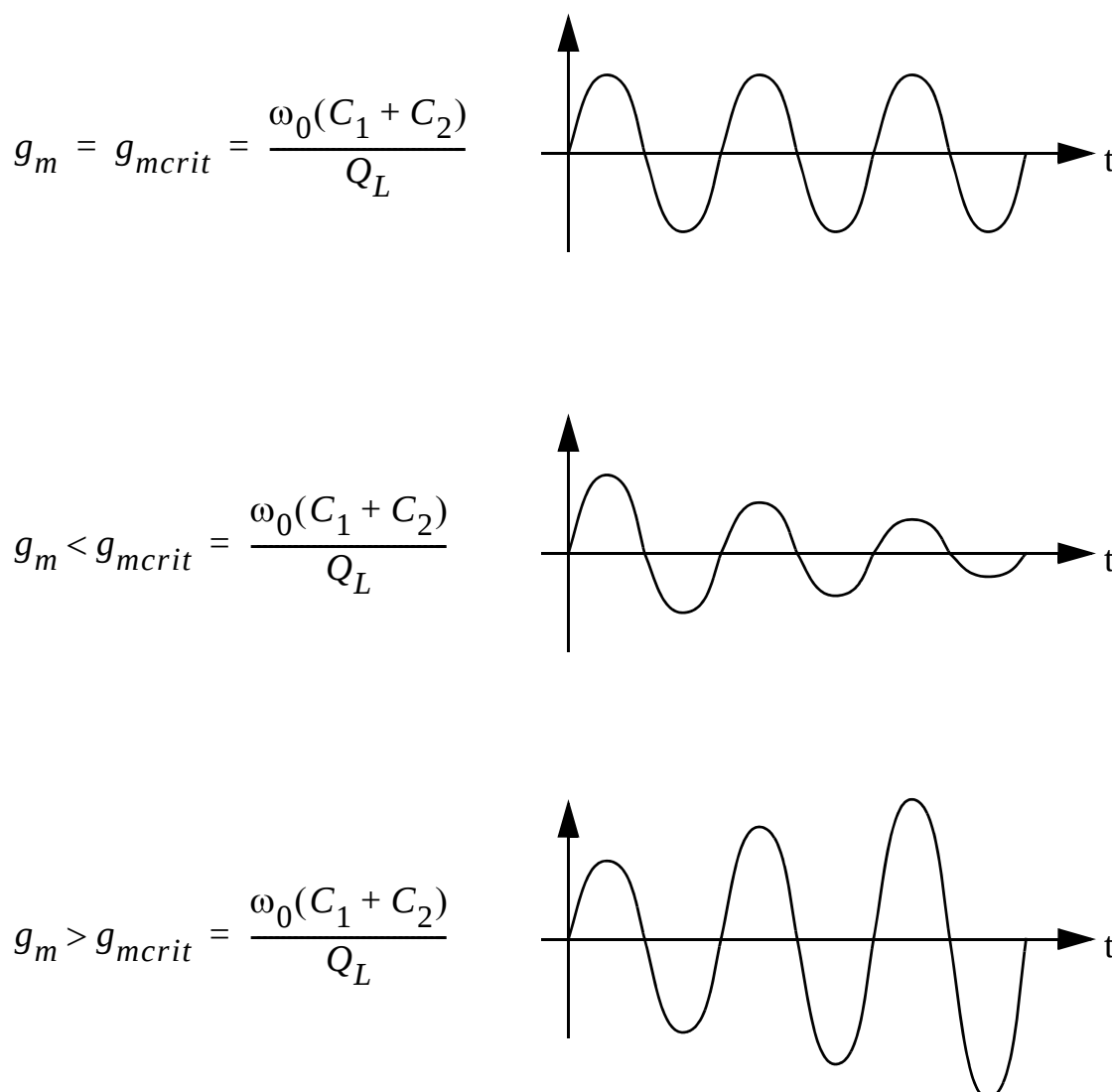
where  $Q_L \equiv (\omega_0 L)/r$  is the unloaded quality factor of the inductor.

## ANALYSIS OF THE PIERCE OSCILLATOR (2/2)

The holding condition for the oscillations is thus given by:

$$g_m = g_{mcrit} = \frac{\omega_0(C_1 + C_2)}{Q_L} \quad (9.14)$$

Eqn. 9.14 shows that the transconductance must be large when the losses in the inductor are large (or the unloaded quality factor is small). For the oscillations to begin, we need  $g_m > g_{mcrit}$ . For  $g_m < g_{mcrit}$  however, the oscillations will be damped (cf Fig. 9-5).



**Fig 9-5:** Conditions for oscillations to begin and be maintained.

## THE EFFECT OF TRANSISTOR CURRENT GAIN ON THE PIERCE OSCILLATOR

If the input impedance of the transistor is not negligible in relation to  $Z_1$  and can be considered purely resistive ( $Z_{in} = r_\pi \equiv \beta/g_m$ ), the oscillation condition (9.7) becomes:

$$g_m + \left( \frac{g_m}{\beta} + j\omega C_1 \right) \cdot [1 + j\omega C_2(r + j\omega L)] + j\omega C_2 = 0 \quad (9.15)$$

or:

$$\begin{cases} g_m \left( 1 + \frac{1}{\beta} \right) - \omega^2 C_2 \left( \frac{g_m}{\beta} L + r C_1 \right) = 0 \\ \frac{g_m r}{\beta} C_2 + C_1 + C_2 - \omega^2 L C_1 C_2 = 0 \end{cases} \quad (9.16)$$

from which we get the oscillation frequency in radians  $\omega_0$ :

$$\omega_0 = 1/\sqrt{L C_{eq}} \quad (9.17)$$

with:  $C_{eq} \equiv \frac{C'_1 C_2}{C'_1 + C_2}$  and  $C'_1 \equiv \frac{C_1}{1 + (g_m r)/\beta} = \frac{C_1}{1 + r/r_\pi}$  (9.18)

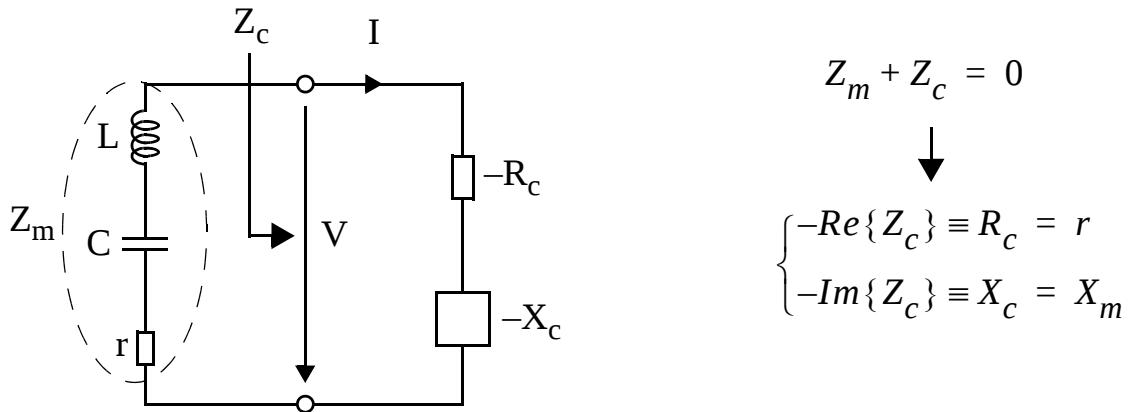
We remark that the losses in the inductor, modeled by the resistance  $r$  associated with the input impedance  $r_\pi$  of the bipolar transistor, noticeably modify the resonant frequency in relation to the ideal case given by (9.12). This effect is negligible as long as  $\beta \gg g_m r$ .

The critical transconductance assuring the continuation of the oscillations is given by:

$$g_{mcrit} = \frac{\omega^2 r C_1 C_2}{1 + \frac{1}{\beta}(1 - \omega^2 L C_2)} = \frac{\omega^2 r C_1 C_2}{1 - \frac{\alpha}{\beta} \left( 1 + \frac{g_m r}{\beta} \right)} \cong \frac{\omega(C_1 + C_2)}{Q_L} \quad (9.19)$$

for  $\beta \gg \alpha$  and  $\beta \gg g_m r$  where  $\alpha \equiv C_2/C_1$ .

## ANALYSIS BY THE NEGATIVE RESISTANCE METHOD



**Fig 9-6:** *Resonant circuit in which the losses are compensated by a negative resistance.*

Once excited, an ideal resonant circuit (meaning without losses), will continue to oscillate indefinitely. In reality, the oscillations will dampen until they disappear due to the dissipation of energy in the resistances. The role of the active element is precisely to provide the energy necessary to maintain the oscillations and thus to compensate the energy dissipated in the resistance  $r$ , representing the losses of the resonant circuit. This source of energy can be interpreted as a negative resistance. As long as the total series resistance of the circuit seen by the resonant circuit in Fig. 9-6 is negative, the oscillations will grow, whereas they will be attenuated as long as this resistance remains positive. The amplitude of the oscillations is maintained when the negative resistance exactly compensates the loss resistance. The critical oscillation condition is thus expressed by:

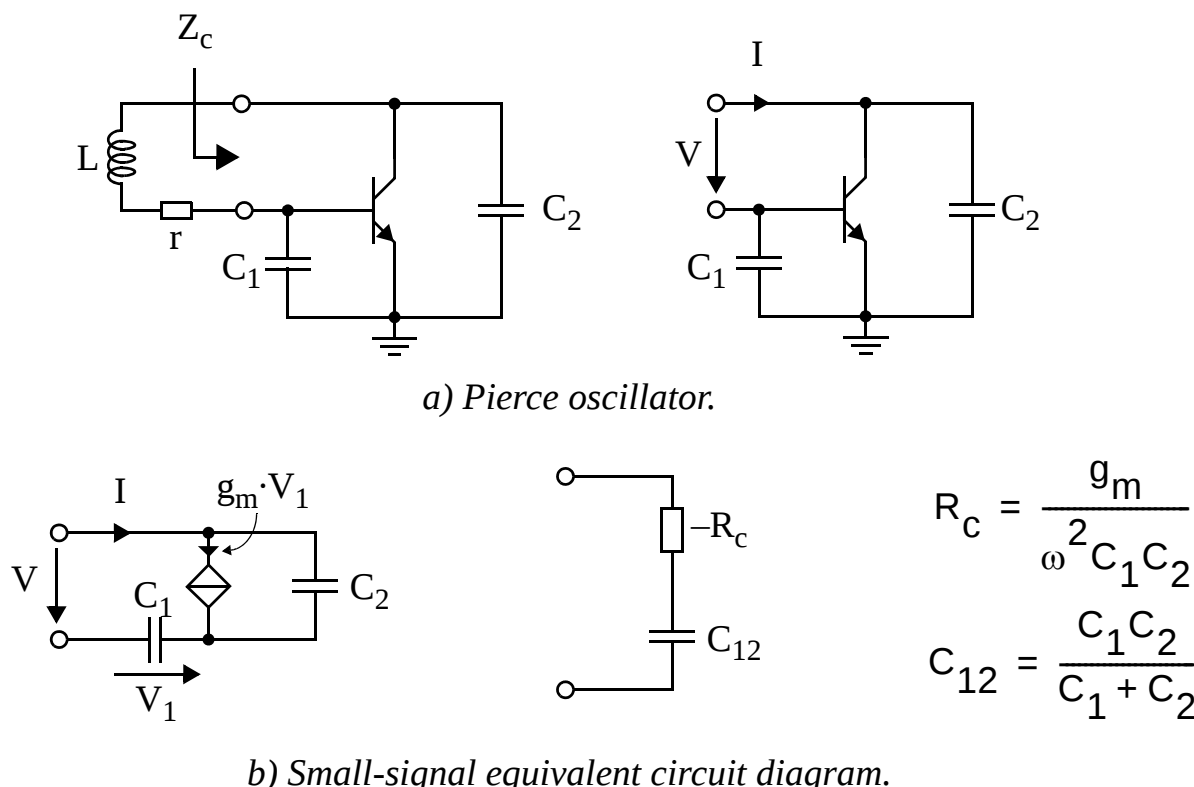
$$Z_m(\omega) + Z_c(\omega, g_m) = 0 \quad (9.20)$$

or:

$$\begin{cases} -\operatorname{Re}\{Z_c\} \equiv R_c(\omega, g_m) = r \\ -\operatorname{Im}\{Z_c\} \equiv X_c(\omega, g_m) = X_m(\omega) \end{cases} \quad (9.21)$$

where  $X_m$  is the reactance of the resonant circuit.

## LINEAR ANALYSIS OF THE PIERCE OSCILLATOR BY THE NEGATIVE RESISTANCE METHOD (1/3)



**Fig 9-7:** Negative resistance of the Pierce oscillator.

This approach can be applied to the Pierce oscillator shown in Fig. 9-7 a). In this case the resonant RLC circuit in Fig. 9-6 reduces to an inductance in series with the loss resistance. The series capacitance forming the series resonant circuit of Fig. 9-6 is thus included in the impedance  $Z_c$  of the active circuit. The small-signal equivalent diagram is shown in Fig. 9-7 b) where we have assumed that  $(g_m/\beta) \ll \omega_0 C_1$ . The input impedance of this circuit is then:

$$Z_c \equiv \frac{V}{I} = - \frac{g_m + j\omega(C_1 + C_2)}{\omega^2 C_1 C_2} = - \frac{g_m}{\omega^2 C_1 C_2} + \frac{1}{j\omega C_{12}} = -R_c - jX_c \quad (9.22)$$

## LINEAR ANALYSIS OF THE PIERCE OSCILLATOR BY THE NEGATIVE RESISTANCE METHOD (2/3)

The oscillation frequency is then given by the condition on the imaginary part:

$$-Im\{Z_c\} \equiv X_c = X_m \Rightarrow \frac{1}{\omega C_{12}} = \omega L \quad (9.23)$$

from which we get:

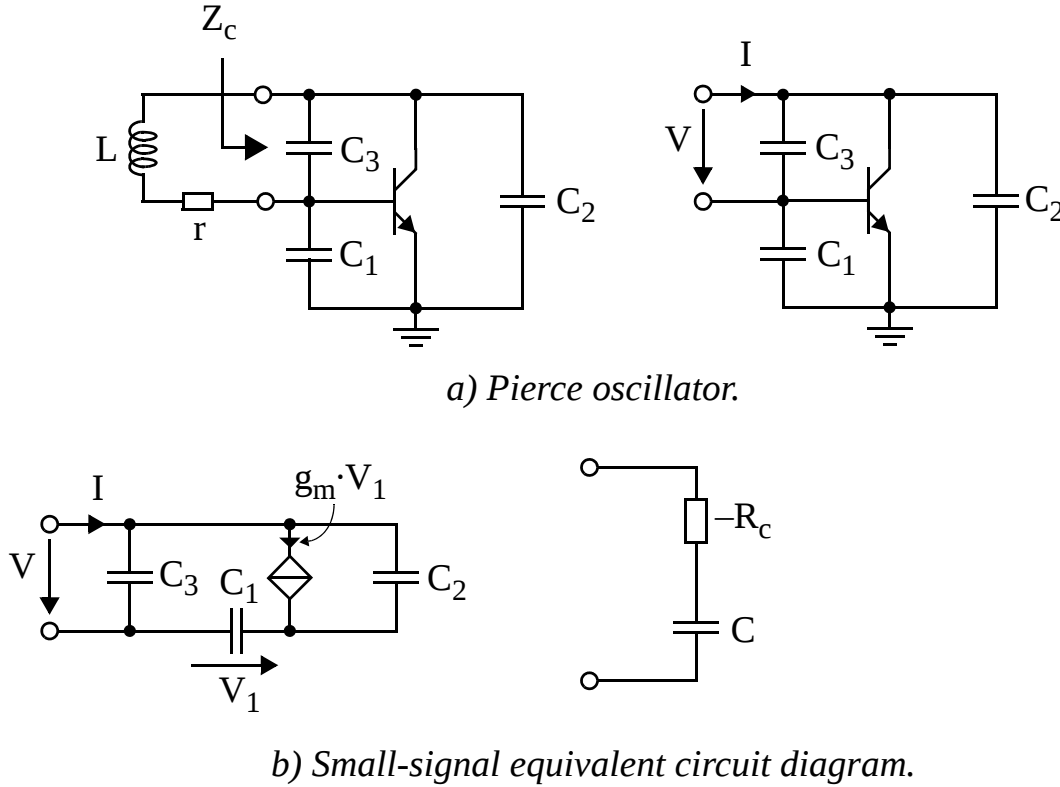
$$\omega_0 = \frac{1}{\sqrt{LC_{12}}} \quad \text{with:} \quad C_{12} \equiv \frac{C_1 C_2}{C_1 + C_2} \quad (9.24)$$

When the inductor is connected, the circuit will oscillate if the resistance  $R_c$  is equal to the loss resistance of the inductor  $r$ , from which we get the following oscillation condition:

$$g_m = g_{mcrit} \equiv r \cdot \omega_0^2 C_1 C_2 = \frac{r}{L} (C_1 + C_2) = \frac{\omega_0 (C_1 + C_2)}{Q_L} \quad (9.25)$$

We find once again the result (9.13) previously obtained by applying the Barkhausen criterion.

## LINEAR ANALYSIS OF THE PIERCE OSCILLATOR BY THE NEGATIVE RESISTANCE METHOD (3/3)



**Fig 9-8:** *Negative resistance of the Pierce oscillator.*

A more careful analysis can be carried out using the diagram in Fig. 9-8 where the capacitor  $C_3$  has been added in order to take the parasitic capacitance associated with the inductor and the capacitance  $C_{bc}$  of the bipolar transistor into account. The small-signal equivalent circuit diagram in Fig. 9-8 b) lets us calculate the impedance  $Z_c$ :

$$Z_c = - \frac{g_m + j\omega(C_1 + C_2)}{\omega^2(C_1C_2 + C_1C_3 + C_2C_3) - j\omega g_m C_3} \quad (9.26)$$

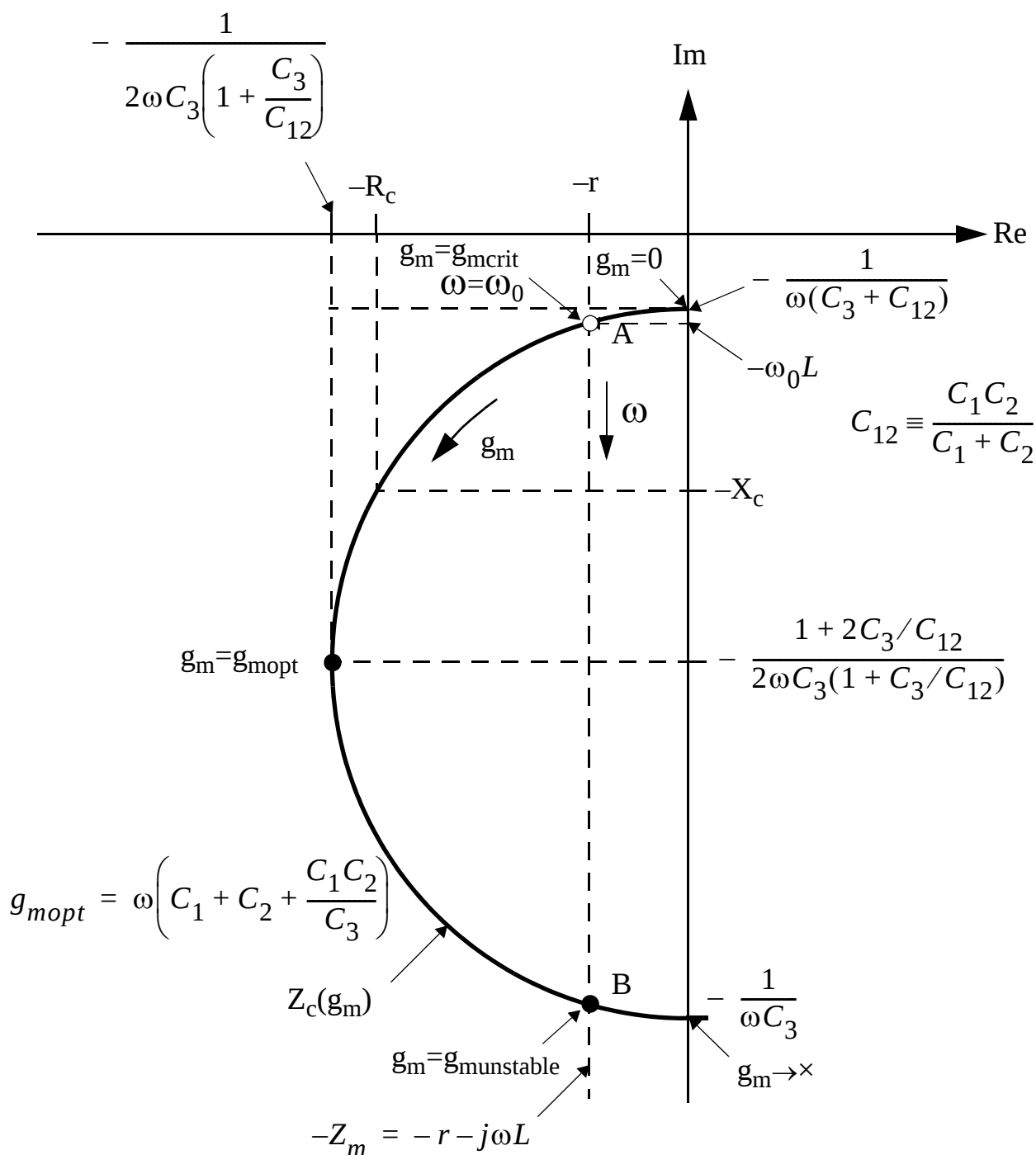
From which we get:

$$R_c = \frac{g_m C_1 C_2}{(g_m C_3)^2 + \omega^2(C_1C_2 + C_1C_3 + C_2C_3)^2} \quad (9.27)$$

$$X_c = \frac{g_m^2 C_3 + \omega^2(C_1 + C_2)(C_1C_2 + C_1C_3 + C_2C_3)}{\omega[(g_m C_3)^2 + \omega^2(C_1C_2 + C_1C_3 + C_2C_3)^2]} \quad (9.28)$$

## IMPEDANCE POINTS OF THE ACTIVE CIRCUIT

An effective representation that allows a good comprehension of the oscillator and simplifies its dimensioning, is obtained by plotting the points corresponding to the impedance  $Z_c(g_m)$  in the complex plane as a function of the transconductance. As indicated in Fig. 9-9, we obtain a semi-circle centered on the imaginary axis and entirely contained in the 3<sup>rd</sup> quadrant.



**Fig 9-9:** Representation of the oscillator in the complex plane.

## EXISTENCE CONDITION OF A STABLE POINT

We can also plot the impedance points  $-Z_m(\omega)$  as a function of the frequency in radians. For an inductance with losses, we obtain a straight line parallel to the imaginary axis passing through the point on the x-axis  $-r$ . The critical oscillation condition then corresponds to the intersections A and B of the straight line and the semi-circle. We can show that only the point A corresponds to stable oscillations. The oscillation frequency in radians  $\omega_0$  and the critical transconductance  $g_{mcrit}$  corresponding to the stable point A can be found by resolving the system of equations (9.21) with  $X_m = \omega L$  as a function of  $\omega$  and  $g_m$ , where  $R_c$  and  $X_c$  are given by (9.27) and (9.28).

The negative resistance  $-R_c$  of the circuit attains a minimum (maximum in absolute value):

$$R_{c-max} = \frac{1}{2\omega C_3 \left(1 + \frac{C_3}{C_{12}}\right)} \quad (9.29)$$

for a specific value  $g_{mopt}$  of the transconductance:

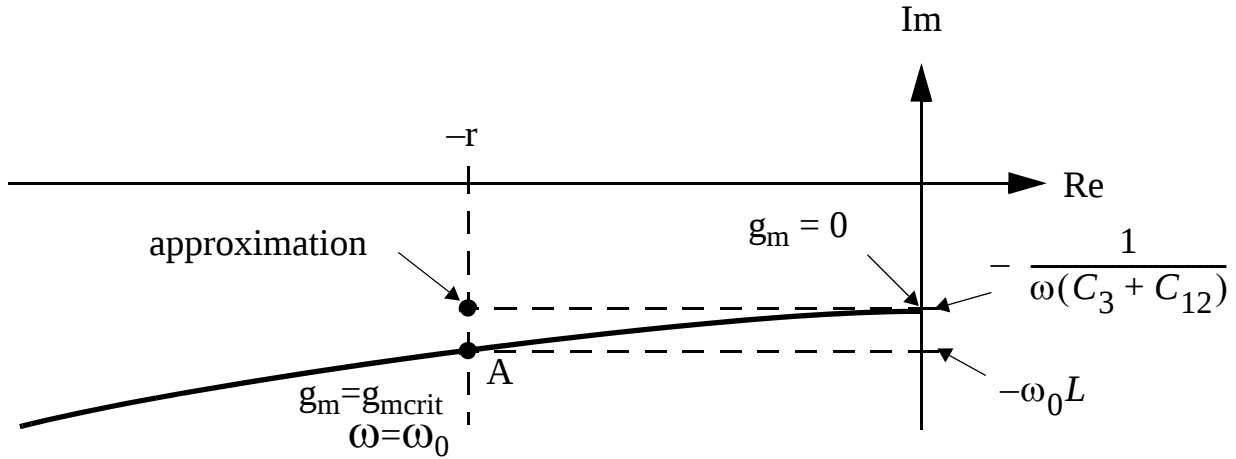
$$g_{mopt} = \omega \left( C_1 + C_2 + \frac{C_1 C_2}{C_3} \right) \quad (9.30)$$

If the loss resistance of the inductor  $r$  is greater than  $R_{c-max}$  there is no longer an intersection, and oscillations are no longer possible no matter what the value of the transconductance. The existence condition of a solution can thus be expressed as:

$$\frac{1}{2\omega C_3 \left(1 + \frac{C_3}{C_{12}}\right)} \geq r \quad (9.31)$$

## APPROXIMATION OF THE OSCILLATION FREQUENCY

The representation in Fig. 9-9 also allows us to qualitatively evaluate the influence of a variation in the capacitances  $C_i$  ( $i=1,2,3$ ) on the oscillation frequency. For example, when  $C_1$  and  $C_2$  decrease, the radius of the circle also decreases, and the point A moves downward, which has the effect of increasing  $\omega_0$ . When  $C_3 = 0$ , the semi-circle reduces to a horizontal straight line, and  $\omega_0$  no longer depends on  $g_m$ . In general, the oscillation frequency is a function of  $r$  and thus of the quality factor  $Q_L$  of the inductor. This is not good, because this quality factor can vary greatly, causing a large dispersion of the oscillation frequency.



**Fig 9-10:** Approximation of the oscillation frequency.

If the losses in the inductor are small (and thus the quality factor  $Q_L$  is high), the vertical straight line in Fig. 9-9 approaches the imaginary axis. The sensitivity of  $\omega_0$  to  $r$  and thus to  $Q_L$  then becomes small. In these conditions, we can approximate the circle by the tangent to the point  $g_m = 0$ . An approximate value of  $\omega_0$  is thus given by setting  $g_m = 0$  in the term  $X_c(\omega, g_m)$  of (9.21) and solving for  $\omega_0$ . We then find:

$$\omega_0 \cong \frac{1}{\sqrt{L(C_3 + C_{12})}} \quad \text{with:} \quad C_{12} \equiv \frac{C_1 C_2}{C_1 + C_2} \quad (9.32)$$

## APPROXIMATION OF THE CRITICAL TRANSCONDUCTANCE

For the same conditions, an approximation of the critical transconductance can be found by solving  $R_c(\omega, g_m) = r$  where  $R_c$  is given by (9.27) and  $\omega$  by the approximation (9.32). We then find:

$$g_{mcrit} \cong \frac{1}{r} \cdot \frac{\alpha_1}{2\alpha_3^2} \left[ 1 - \sqrt{1 - \left( \frac{2\alpha_3(\alpha_1 + 1)}{\alpha_1 Q_L} \right)^2} \right] \quad (9.33)$$

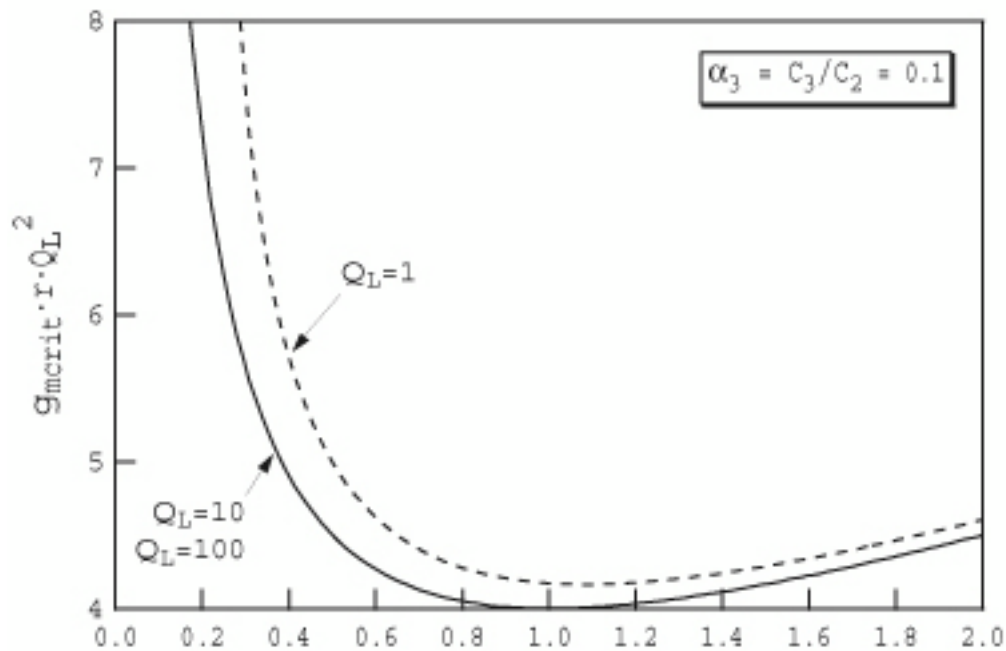
$$\cong \frac{1}{r} \cdot \frac{(\alpha_1 + 1)^2}{\alpha_1 Q_L^2} = \frac{\omega_0}{Q_L} \cdot (C_1 + C_2) \left( 1 + \frac{C_3}{C_{12}} \right)$$

where  $\alpha_1 \equiv C_1/C_2$ ,  $\alpha_3 \equiv C_3/C_2$  and  $Q_L \equiv (\omega_0 L)/r$ . Eqn. 9.33 only has a solution for  $(2\alpha_3(\alpha_1 + 1))/(\alpha_1 Q_L) < 1$  and thus for

$$Q_L \geq 2\alpha_3 \left( 1 + \frac{1}{\alpha_1} \right) = 2 \frac{C_3}{C_{12}} \quad (9.34)$$

We again find the existence condition (9.31) where  $\omega$  is replaced by (9.32).

## MINIMUM CRITICAL TRANSCONDUCTANCE

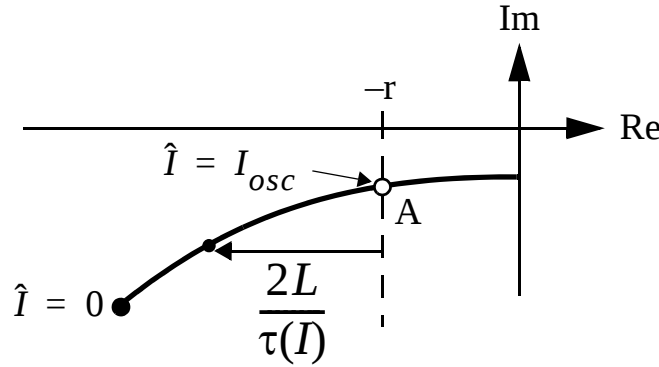


**Fig 9-11:**  $g_{m\text{crit}}$  as a function of the ratio  $\alpha_1 \equiv C_1/C_2$ .

Fig. 9-11 shows that  $g_{m\text{crit}}$  is minimum for  $\alpha_1 = 1$ , meaning for  $C_1 = C_2$  as long as  $Q_L \gg 2\alpha_3$ . The minimum value of  $g_{m\text{crit}}$  is thus given by:

$$g_{m\text{crit-min}} = \frac{1}{r} \cdot \left( \frac{2}{Q_L} \right)^2 = \frac{\omega_0}{Q_L} \cdot 2(C_1 + 2C_3) \quad (9.35)$$

## OSCILLATION START-UP TIME



**Fig 9-12:** *Start-up of oscillations.*

As soon as  $-Re\{Z_c\} \equiv R_c > r$ , the oscillations begin to increase exponentially, starting from the noise, with a time constant given by:

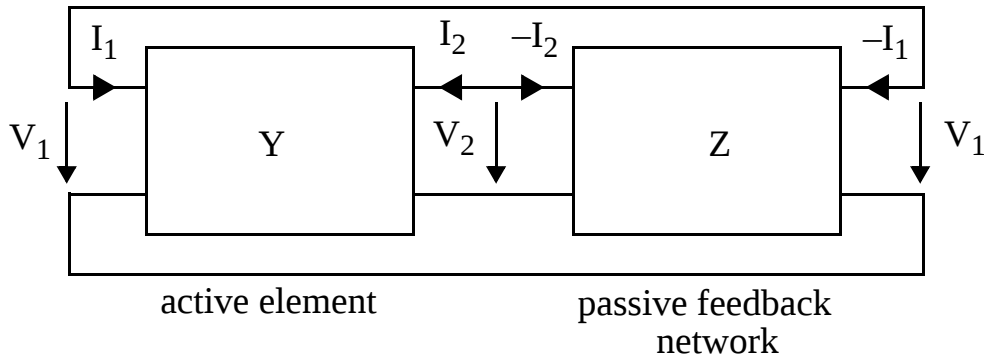
$$\tau = -\frac{L}{r + Re\{Z_c\}} = \frac{L}{R_c - r} \quad (9.36)$$

where  $R_c$  corresponds to a certain value of the transconductance  $g_m > g_{mcrit}$  fixed by the polarization current.

The start-up time is minimum for  $R_c = R_{c-max}$  and thus for  $g_m = g_{mopt}$ . The corresponding time constant is given by:

$$\begin{aligned} \tau_{min} &= \frac{L}{R_{c-max} - r} \cong \frac{L}{R_{c-max}} \\ &= 2\omega_0 L C_3 \left(1 + \frac{C_3}{C_{12}}\right) = \frac{2C_3}{\omega_0 C_{12}} \end{aligned} \quad (9.37)$$

## ANALYSIS USING Y AND Z PARAMETERS



**Fig 9-13:** *General diagram of an oscillator.*

The validity of the methods of analysis presented until now is limited to simple circuits and to low frequency, because they do not take into account all the parasitic elements, especially those associated with the active element. Often this element is simply characterized in terms of its  $S$  or  $Y$  parameters for a given operating point. An oscillator functioning in the small-signal regime can be modeled by the general diagram in Fig. 9-13 where the active component is represented by its admittance matrix  $Y$  and the feedback network by its impedance matrix  $Z$ . From the definitions of the admittance and impedance matrices we have:

$$\begin{aligned} I_1 &= y_{11}V_1 + y_{12}V_2 & \text{and:} & & V_2 &= -z_{11}I_2 - z_{12}I_1 \\ I_2 &= y_{21}V_1 + y_{22}V_2 & & & V_1 &= -z_{21}I_2 - z_{22}I_1 \end{aligned} \quad (9.38)$$

We can show that the condition whereby the voltages  $V_1$  and  $V_2$  are non-zero, and there are oscillations, is given by:

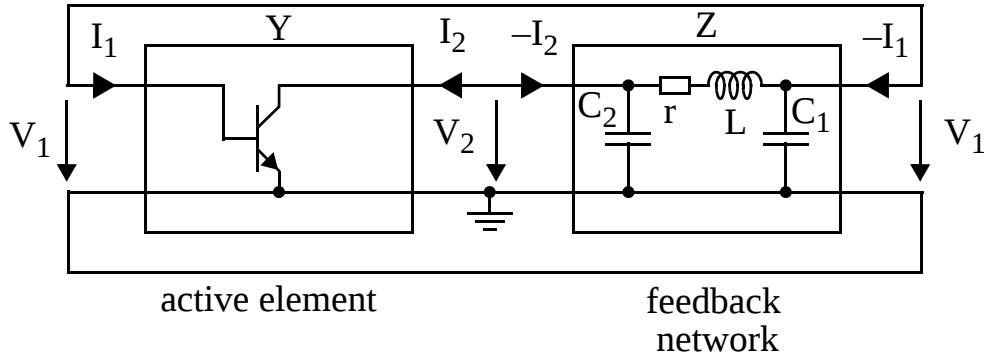
$$\boxed{y_{21}z_{21} + y_{11}z_{22} + y_{22}z_{11} + y_{12}z_{12} + \Delta Y \Delta Z + 1 = 0} \quad (9.39)$$

with:

$$\begin{aligned} \Delta Y &\equiv y_{11}y_{22} - y_{21}y_{12} \\ \Delta Z &\equiv z_{11}z_{22} - z_{21}z_{12} \end{aligned} \quad (9.40)$$

The oscillation condition (9.39) is general and can thus be applied to practically any oscillator.

## EXAMPLE OF THE PIERCE OSCILLATOR (1/2)



**Fig 9-14:** General diagram of an oscillator.

The Pierce oscillator presented in Fig. 9-4 was redrawn in Fig. 9-14, where the transistor is simply represented by its admittance matrix. The impedance matrix of the feedback circuit can be calculated by applying the definitions. We find:

$$\begin{aligned} z_{11} &= -(X_1 X_2 + X_2 X_3 - jr X_2) / Z \\ z_{22} &= -(X_1 X_2 + X_1 X_3 - jr X_1) / Z \\ z_{21} &= z_{12} = -X_1 X_2 / Z \end{aligned} \quad (9.41)$$

where:  $Z \equiv r + j(X_1 + X_2 + X_3)$  (9.42)

From which we get:  $\Delta Z = -(X_1 X_2 / Z)(r + jX_3)$  (9.43)

For the transistor, we set:

$$\begin{aligned} y_{11} &= g_{in} & y_{12} &= jb_{12} \\ y_{21} &= g_m + jb_m & y_{22} &= g_{out} \end{aligned} \quad (9.44)$$

## EXAMPLE OF THE PIERCE OSCILLATOR (2/2)

By plugging (9.41), (9.42) and (9.44) into (9.39), we find:

$$g_m X_1 X_2 = r + K_1 \quad X_1 + X_2 + X_3 = K_2 \quad (9.45)$$

with:

$$\begin{aligned} K_1 &\equiv -g_{in} X_1 (X_2 + X_3) - g_{out} X_2 (X_1 + X_3) \\ &\quad - r X_1 X_2 (g_{in} g_{out} + b_m b_{12}) - g_m b_{12} X_1 X_2 r \\ K_2 &\equiv (b_m + b_{12}) X_1 X_2 + g_{in} g_{out} X_1 X_2 X_3 \\ &\quad - r (g_{in} X_1 + g_{out} X_2) + b_{12} b_m X_1 X_2 X_3 - g_m b_{12} X_1 X_2 r \end{aligned} \quad (9.46)$$

For the bipolar transistor in Fig. 9-4, by ignoring the base current and assuming that the transistor is working at low frequency, we can set  $y_{11} = y_{12} = y_{22} \cong 0$  and  $y_{21} \cong g_m$ . Under these conditions,  $K_1 = K_2 = 0$  and so:

$$g_m X_1 X_2 = r \quad X_1 + X_2 + X_3 = 0 \quad (9.47)$$

By taking  $X_i = -1/(\omega C_i)$  for  $i = 1, 2$  and  $X_3 = \omega L$ , we find:

$$\frac{g_m}{\omega^2 C_1 C_2} = r \quad -\frac{1}{\omega C_1} - \frac{1}{\omega C_2} + \omega L = 0 \quad (9.48)$$

which is just the condition (9.11). By taking the base current into account we will get  $y_{11} \cong g_m/\beta$  and so

$$K_1 = -\frac{g_m}{\beta} X_1 (X_2 + X_3) \quad K_2 = -r \frac{g_m}{\beta} X_1 \quad (9.49)$$

The condition (9.11) then becomes:

$$\begin{aligned} \frac{g_m}{\omega^2 C_1 C_2} &= r + \frac{g_m}{\beta} \frac{1}{\omega C_1} \left( -\frac{1}{\omega C_2} + \omega L \right) \\ -\frac{1}{\omega C_1} - \frac{1}{\omega C_2} + \omega L &= -r \frac{g_m}{\beta} \frac{1}{\omega C_1} \end{aligned} \quad (9.50)$$

which is identical to the condition (9.16).

## QUASI-SINUSOIDAL CONTROL VOLTAGE

If  $g_m > g_{mcrit}$ , oscillations will begin and grow. When their amplitude becomes large, harmonics are generated by the non-linearity of the active element. An oscillator analysis which takes these non-linear characteristics into account is difficult, even impossible. For the Pierce oscillator presented in Fig. 9-8, assuming that the quality factor of the inductor is high ( $Q_L \geq 10$ ), the collector current, which is rich in harmonics, is filtered by the inductance. The current integrated over the capacitance  $C_1$  is then approximately sinusoidal. As long as the capacitance  $C_1$  is linear, the control voltage of the bipolar transistor can also be considered to be sinusoidal:

$$v_{be}(t) = V_{BEq} + \Delta V_{BE} \cdot \cos(\omega_0 t) \quad (9.51)$$

The collector current is then given by:

$$i_c(t) = I_s \cdot e^{\frac{v_{be}(t)}{U_T}} = I_s \cdot e^{\frac{V_{BEq} + \Delta V_{BE} \cdot \cos(\omega_0 t)}{U_T}} = I_q \cdot e^{x \cdot \cos(\omega_0 t)} \quad (9.52)$$

where:

$$I_q \equiv I_s \cdot e^{\frac{V_{BEq}}{U_T}} \quad \text{and} \quad x \equiv \frac{\Delta V_{BE}}{U_T} \quad (9.53)$$

Notice that it is essentially the parasitic capacitance  $C_3$  that will couple harmonic components of the collector current to the capacitance  $C_1$  and add them to the control voltage  $v_{be}(t)$ . The hypothesis of a sinusoidal control voltage is thus only valid when the parasitic capacitance  $C_3$  remains negligible in relation to  $C_{12}$ .

## DECOMPOSITION OF THE COLLECTOR CURRENT INTO A FOURIER SERIES

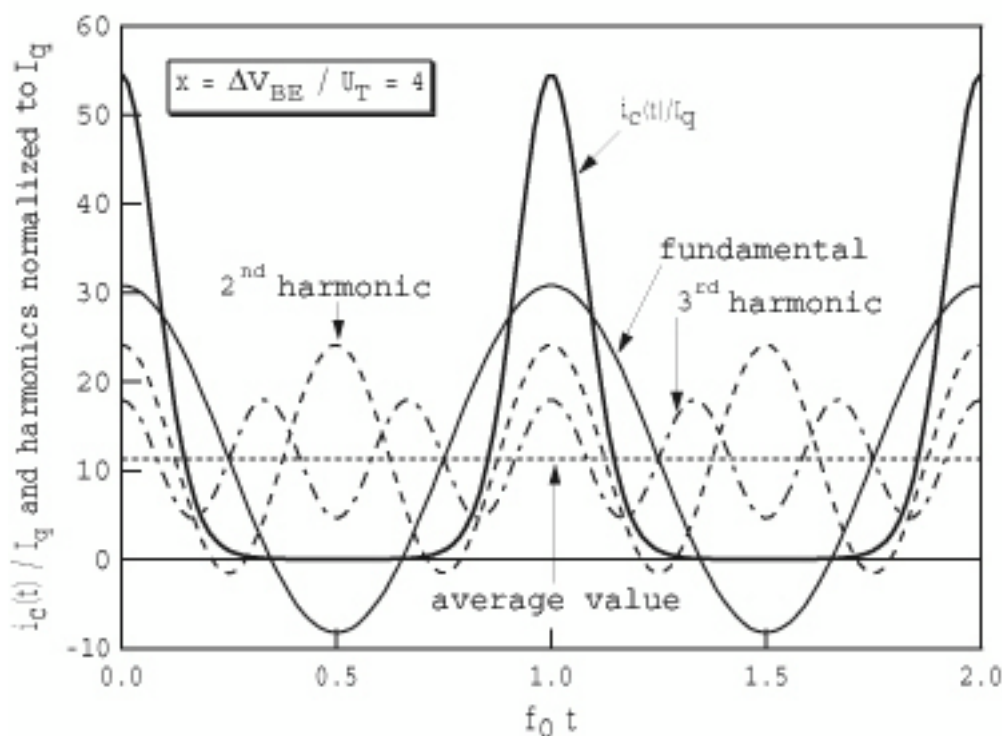
The function  $e^{x \cdot \cos(\omega_0 t)}$  can be expanded into a Fourier series:

$$e^{x \cdot \cos(\omega_0 t)} = I_0(x) + 2 \cdot \sum_{n=1}^{\infty} I_n(x) \cdot \cos(n\omega_0 t) \quad (9.54)$$

where  $I_0(x)$  is the modified Bessel function of order 0 and  $I_n(x)$  are the modified Bessel functions of order  $n$ . The collector current can thus be written:

$$i_c(t) = I_{dc} + 2I_q \cdot \sum_{n=1}^{\infty} I_n(x) \cdot \cos(n\omega_0 t) \quad (9.55)$$

where  $I_{dc}$  represents the average value. The current  $i_c(t)$  normalized to  $I_q$  is represented in Fig. 9-15 for an amplitude of  $4U_T$  with the fundamental component  $2I_0(x) \cdot \cos(\omega_0 t)$ , along with the 2<sup>nd</sup> and 3<sup>rd</sup> harmonics  $2I_n(x) \cdot \cos(n\omega_0 t)$  ( $n = 2, 3$ ).



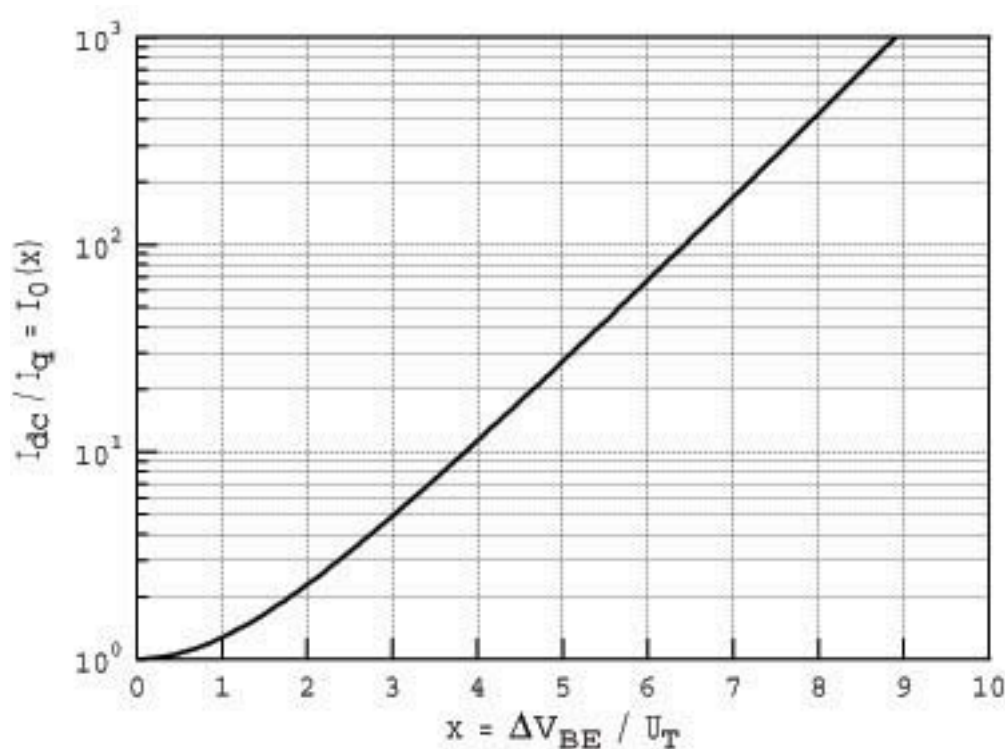
**Fig 9-15:** Collector current for  $x = \Delta V_{BE}/U_T = 4$ .

## AVERAGE VALUE OF THE COLLECTOR CURRENT

Fig. 9-15 also shows the average value of the collector current, which becomes a function of the amplitude of the sinusoidal signal applied between the base and the transmitter:

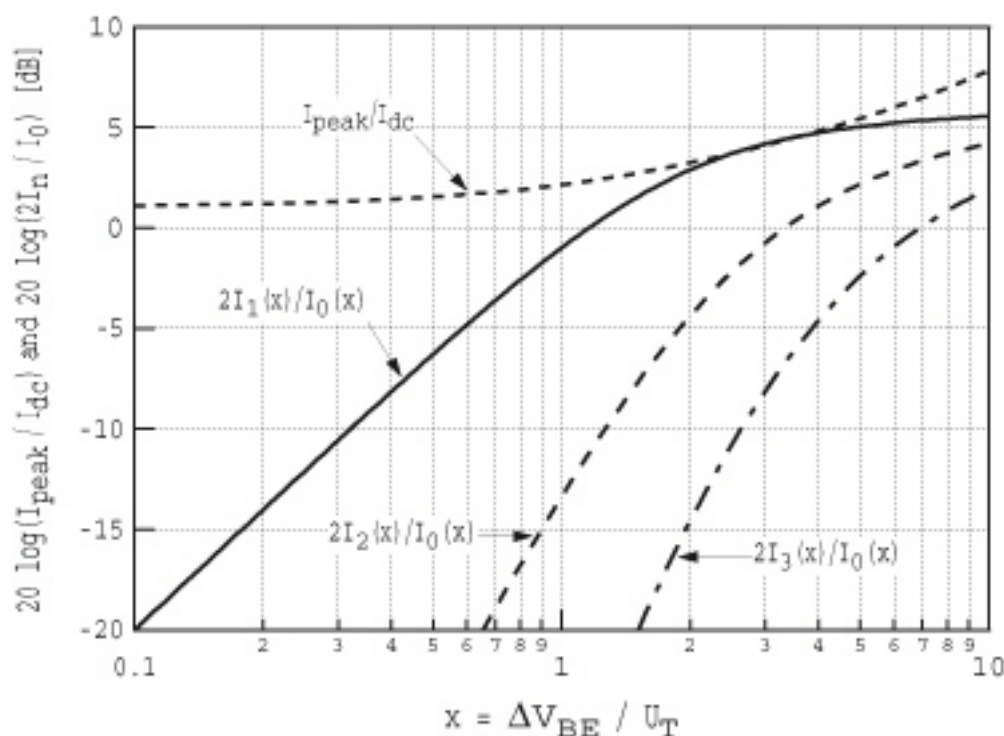
$$I_{dc}(x) = I_q \cdot I_0(x) \cong \begin{cases} I_q & \text{for } x \ll 1 \\ I_q \cdot \frac{e^x}{\sqrt{2\pi x}} & \text{for } x \gg 1 \end{cases} \quad (9.56)$$

As indicated in Fig. 9-16, the average current  $I_{dc}$  increases rapidly with the amplitude of the sinusoidal signal when  $x \geq 1$ .



**Fig 9-16:** Increase of the average current as a function of the normalized amplitude of the sinusoidal signal.

## HARMONIC COMPONENTS OF THE COLLECTOR CURRENT



**Fig 9-17:** *Fundamental component and harmonics as functions of the normalized amplitude of the sinusoidal signal.*

Fig. 9-17 shows the harmonic components of the collector current given by (9.55), normalized to the average value  $I_{dc} = I_q \cdot I_0(x)$ , as well as the normalized peak value:

$$\frac{I_{peak}}{I_{dc}} = \frac{e^x}{I_0(x)} \cong \sqrt{2\pi x} \quad \text{for } x \gg 1 \quad (9.57)$$

We remark that the amplitude of the fundamental grows rapidly and becomes approximately equal to the average value when  $\Delta V_{BE}$  is slightly greater than  $U_T$ . It tends toward an asymptotic value equal to  $2I_{dc}$  (+6 dB in Fig. 9-17).

## EFFECTS OF NON-LINEARITIES OF THE ACTIVE ELEMENT

The active element (often reduced to a transconductance) generally has non-linear characteristics which generate harmonics. If the quality factor of the resonant circuit is high (as for quartz resonators or SAWs), the current  $I$  that traverses the resonant RLC circuit can be considered to be sinusoidal, even if the voltage  $V$  is strongly distorted. Thus, the exchange of energy between the active circuit and the resonator mainly takes place at the fundamental frequency. The non-linear active circuit can thus be replaced by its equivalent impedance  $Z_{c1}$  at the fundamental frequency. This is defined by:

$$Z_{c1} \equiv - \frac{V_1}{I} \quad (9.58)$$

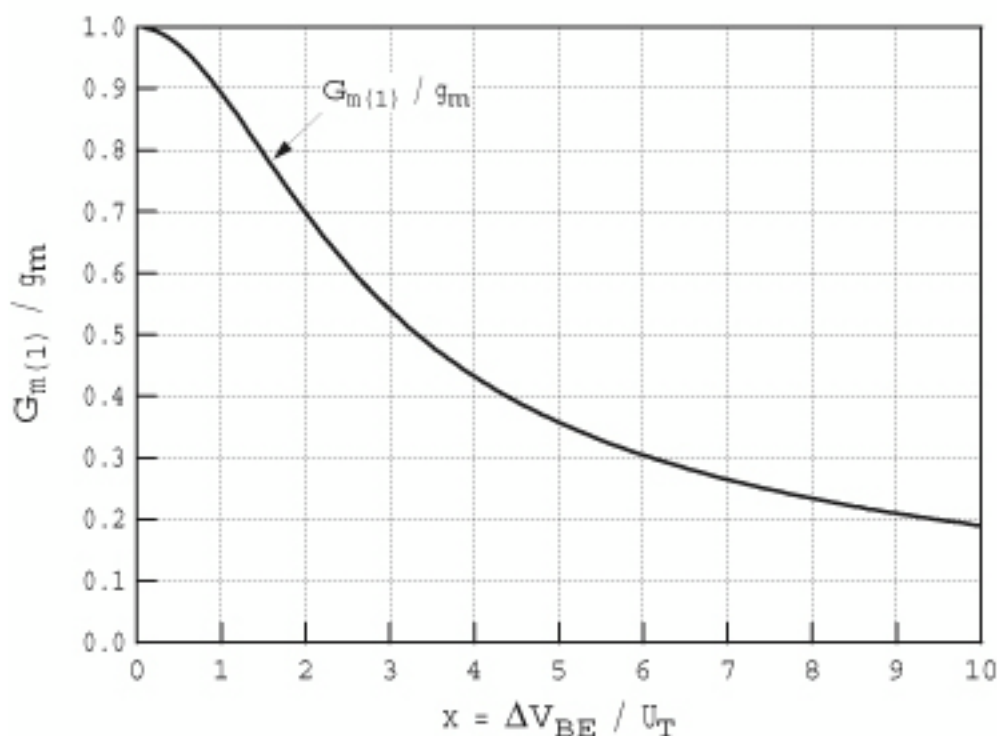
where  $V_1$  is the complex voltage of the fundamental component  $V$ , which depends on the amplitude of the sinusoidal current  $I$ .

## TRANSCONDUCTANCE FOR THE FUNDAMENTAL

At low frequency, the variations in current  $\Delta I_{c(1)}$  and the variations in the control voltage  $\Delta V_{BE}$  for the fundamental frequency are in phase. The small-signal transconductance  $g_m$  can thus be replaced by the (real) transconductance for the fundamental  $G_{m(1)}$  given by:

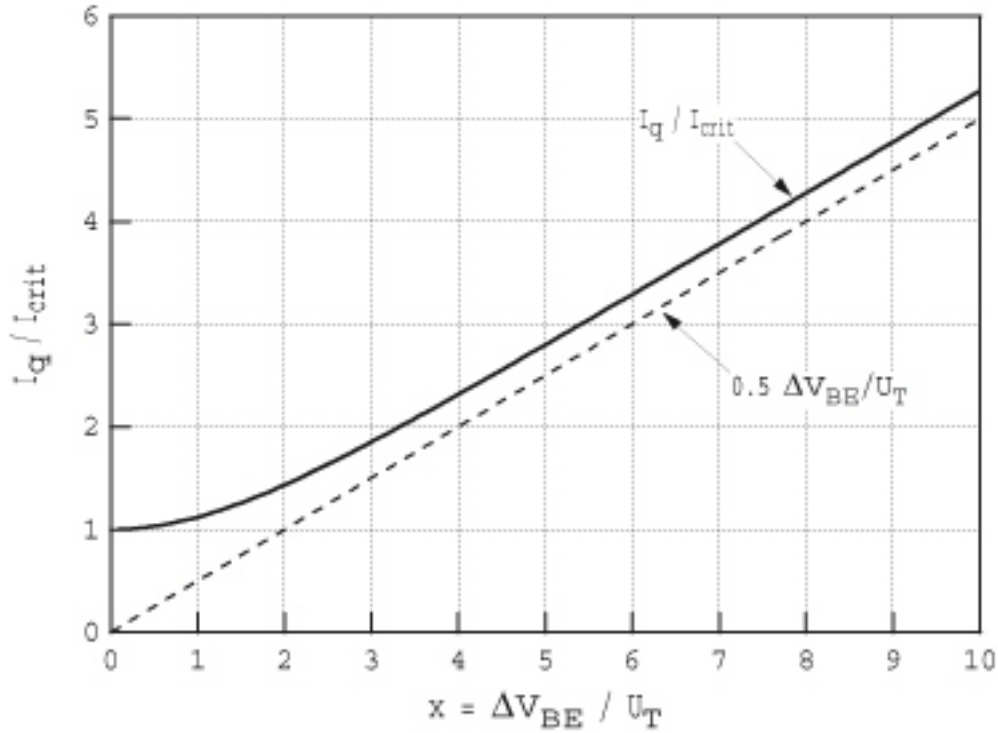
$$G_{m(1)} = \frac{I_q}{V_1} \cdot \frac{2I_1(x)}{I_0(x)} = g_m \cdot \frac{2I_1(x)}{x \cdot I_0(x)} \quad (9.59)$$

where the index (1) refers to the fundamental and  $g_m \equiv I_q/U_T$  is the transconductance without an applied signal ( $x = 0$ ). The function  $G_{m(1)}/g_m$  is represented in Fig. 9-18 as a function of the normalized amplitude  $x \equiv \Delta V_{BE}/U_T$ . We remark that  $G_{m(1)}$  decreases rapidly to attain about 50% of the maximum value obtained in the small-signal regime for an amplitude of  $3.5 \cdot U_T$  or around 90 mV.



**Fig 9-18:** Transconductance for the fundamental as a function of the amplitude of the oscillations.

## LIMITATION OF THE AMPLITUDE



**Fig 9-19:** Polarization current normalized to the critical current as a function of the amplitude of the oscillations.

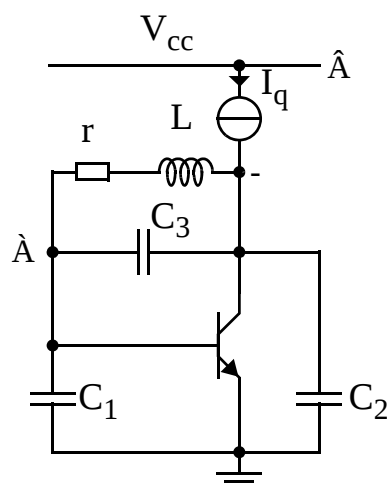
The amplitude of the oscillations will stabilize at a certain value for which  $G_{m(1)} = g_{mcrit}$ . From this equality we get the polarization current  $I_q$  necessary to obtain a certain oscillation amplitude:

$$I_q = I_{crit} \cdot \frac{x \cdot I_0(x)}{2I_1(x)} \quad (9.60)$$

where  $I_{crit} \equiv g_{mcrit} \cdot U_T$ . The current  $I_q$  normalized to  $I_{crit}$  is represented as a function of the normalized amplitude  $\Delta V_{BE} / U_T$  in Fig. 9-19. We remark that for large amplitudes, this currents tends toward the asymptotic value:

$$I_q \cong I_{crit} \cdot \frac{\Delta V_{BE}}{2U_T} = g_{mcrit} \cdot \frac{\Delta V_{BE}}{2} \quad \text{for } \Delta V_{BE} \gg U_T \quad (9.61)$$

## EXAMPLE (1/4)



$$f_0 = 10 \text{ MHz}$$

$$\Delta V_1 = 100 \text{ mV}$$

$$Q_L = 10$$

$$C_3 = 1 \text{ pF}$$

**Fig 9-20:** Example of the dimensioning of a Pierce oscillator.

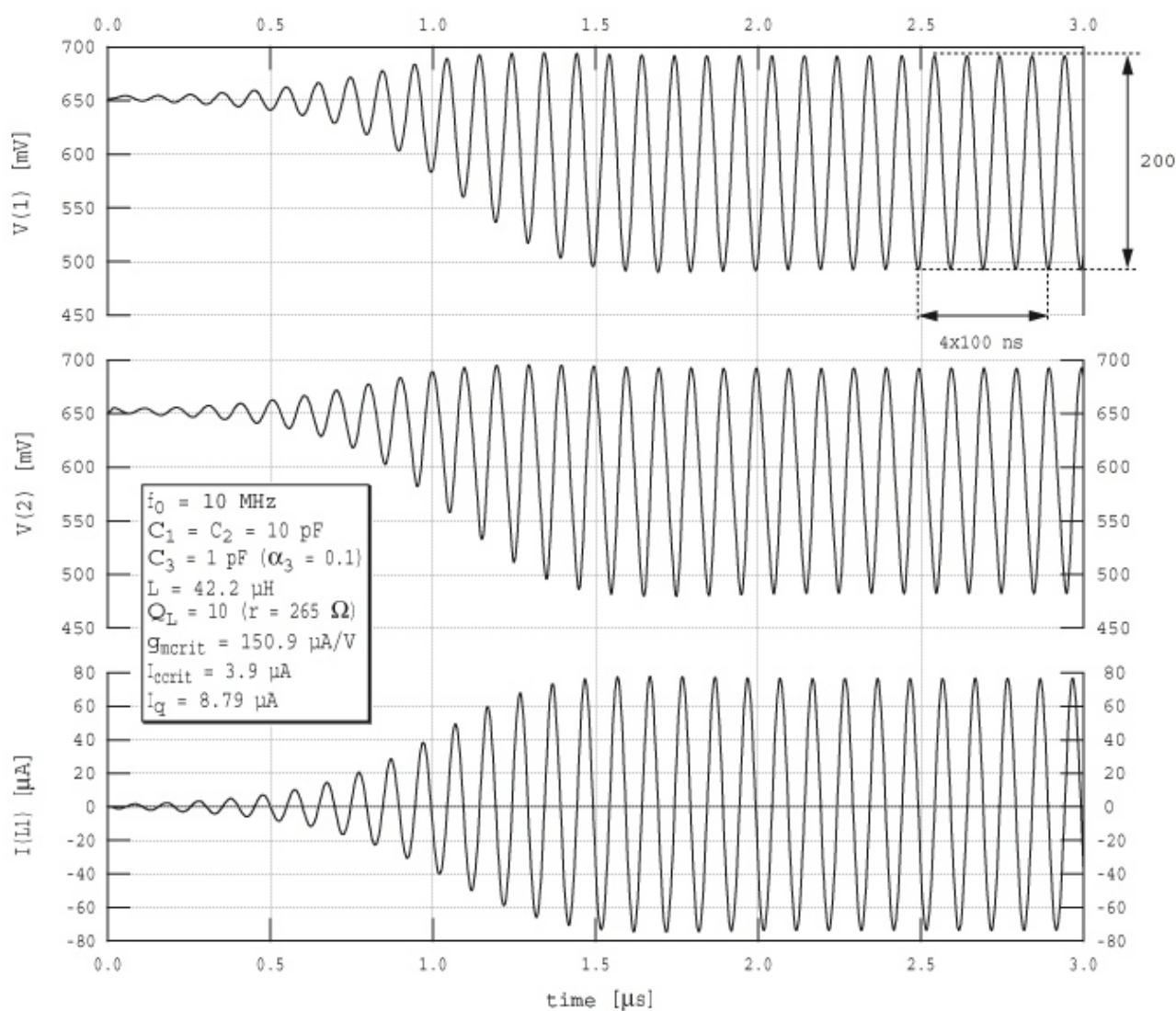
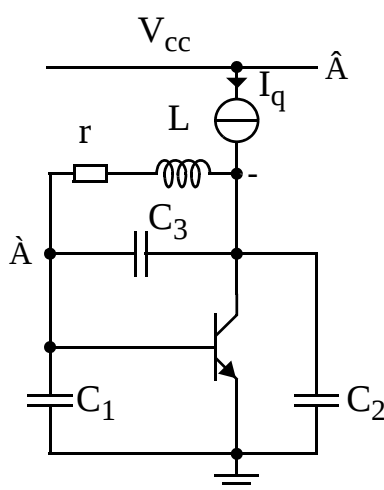
We would like to realize a Pierce oscillator according to the diagram and specifications of Fig. 9-20. Knowing that the transconductance and thus the energy consumption is minimum for  $C_1 = C_2$  and that these capacitances must be at least one order of magnitude greater than the parasitic capacitances (in particular  $C_3 = 1 \text{ pF}$ ) in order to be able to control the oscillation frequency, we take  $C_1 = C_2 = 10 \text{ pF}$ . So  $C_{12} = 5 \text{ pF}$ . From (9.32) we have

$$L = \frac{1}{(2\pi f_0)^2(C_3 + C_{12})} = 42.22 \text{ } \mu\text{H}$$

The loss resistance of the inductance is given by:  $r = (2\pi f_0 L)/Q_L = 265.26 \Omega$ .  $g_{mcrit}$  is given by (9.33) and is  $g_{mcrit} = 150.86 \frac{\mu\text{A}}{\text{V}}$ , from which we get the critical current  $I_{crit} = 3.9 \mu\text{A}$ . The effective transconductance  $G_{m(1)}$  and the polarization current  $I_q$  in order to have an oscillation amplitude of  $\Delta V_1 = 100 \text{ mV}$  can thus be deduced from (9.60) with  $x = \Delta V_1/U_T = 3.865$ . We find  $I_q = 8.79 \mu\text{A}$ .

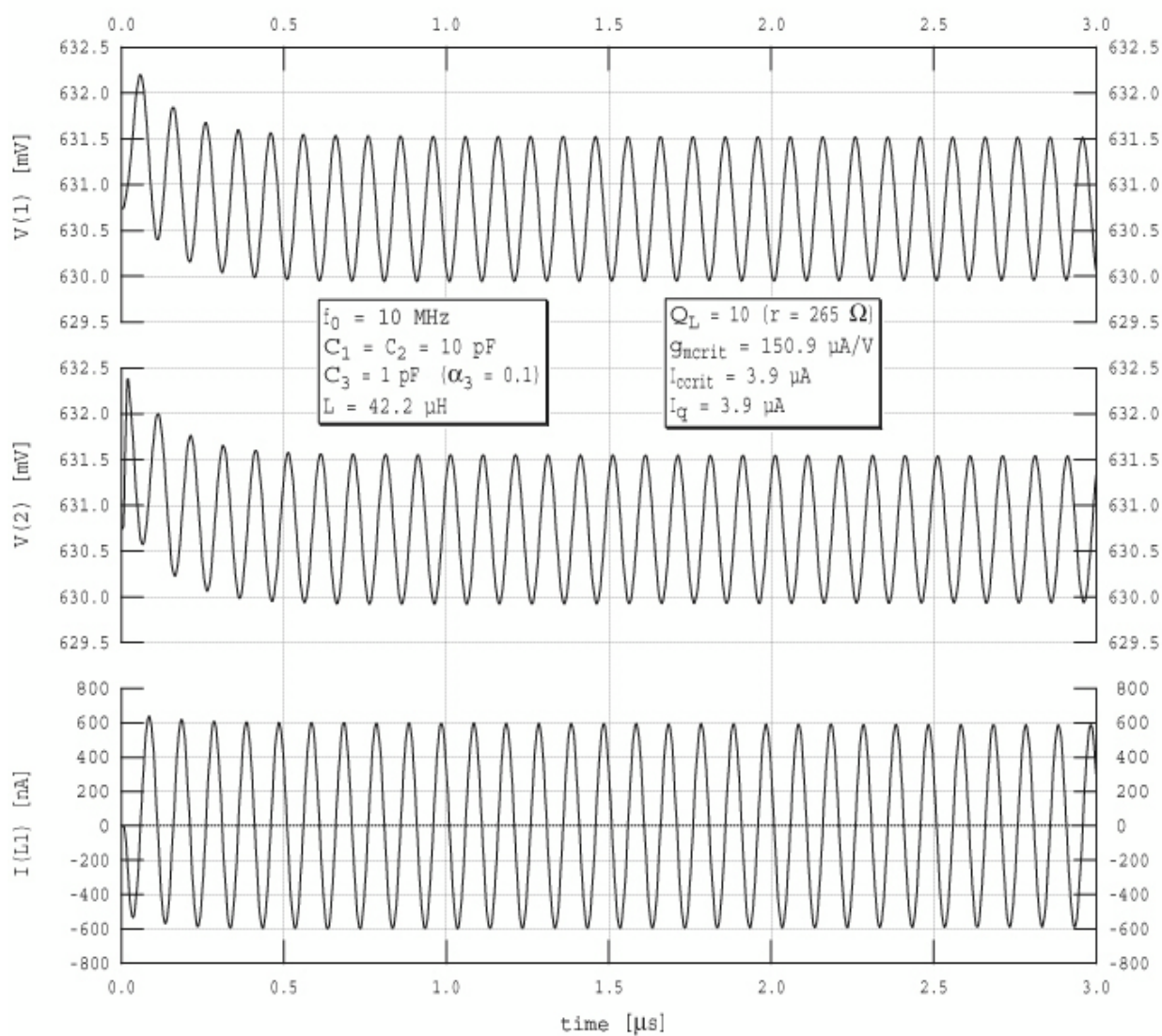
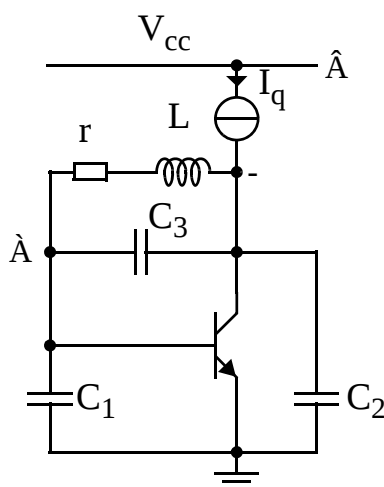
The circuit above has been simulated with Spice and the simulation results are given in Fig. 9-21.

## EXAMPLE (2/5)

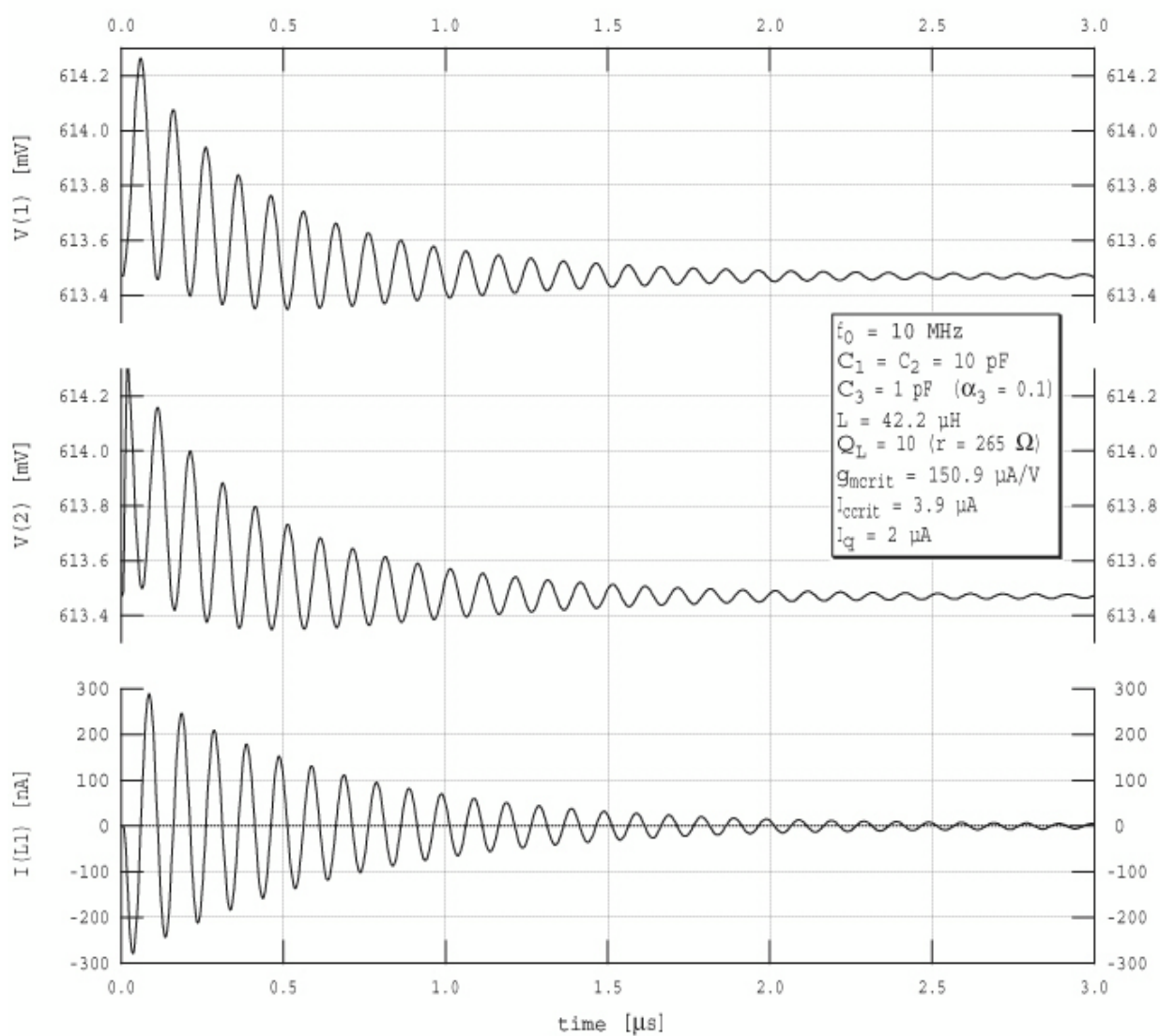
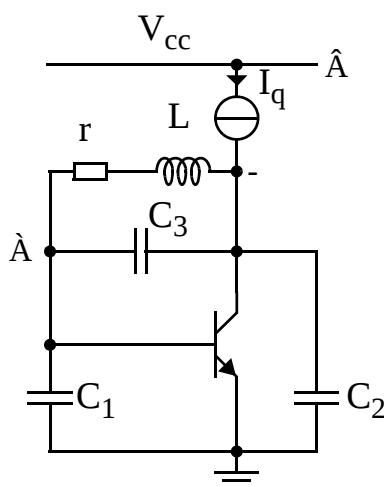


**Fig 9-21:** The supercritical condition with  $I_q > I_{ccrit}$ .

## EXAMPLE (3/5)



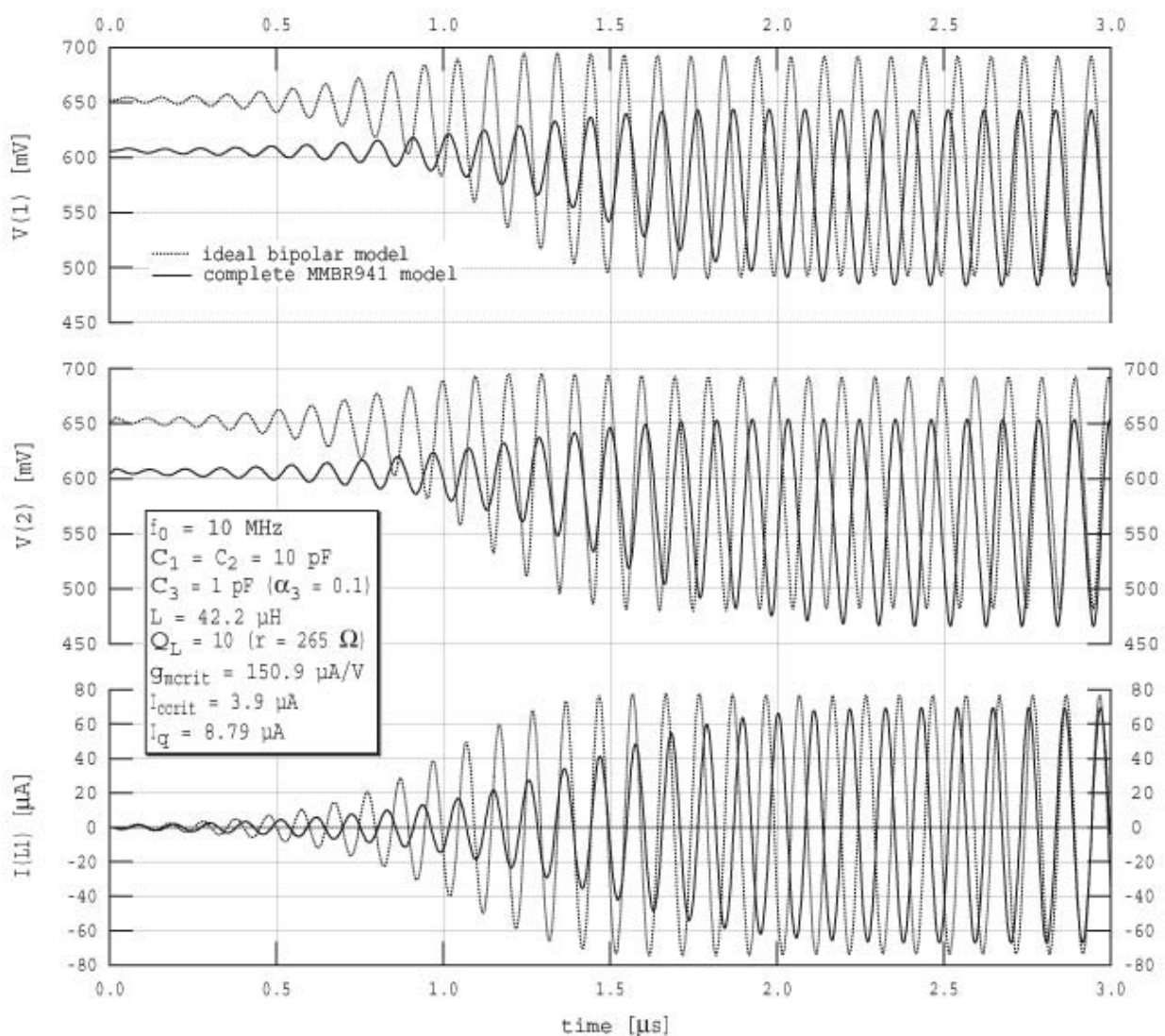
**Fig 9-22:** The critical condition with  $I_q = I_{\text{ccrit}}$ .

**EXAMPLE (4/5)**

**Fig 9-23:** The subcritical condition with  $I_q < I_{\text{ccrit}}$ .

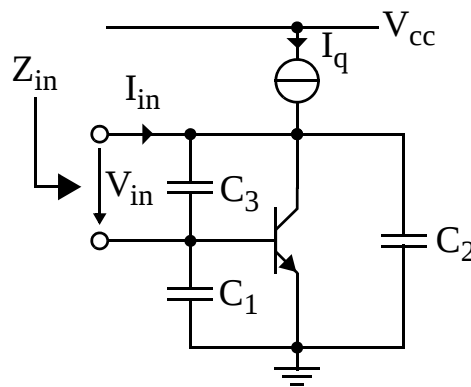
## EXAMPLE (5/5)

Fig. 9-24 shows the simulation results obtained by replacing the ideal transistor of Fig. 9-20 by the MMBR941 used in Chapter 7 (cf Fig. 7-12). We notice that the simulation is not very different from that obtained with the ideal transistor. Specifically, the oscillation frequency is still 10 MHz, but the amplitudes at nodes 1 and 2 are smaller (75 mV for V(1) and 90 mV for V(2) instead of 100 mV).



**Fig 9-24:** Simulation with the transistor MMBR941.

## NON-LINEAR EFFECTS (1/3)



**Fig 9-25:** Circuit used for the simulation of the impedance for the fundamental  $Z_{in(1)}$ .

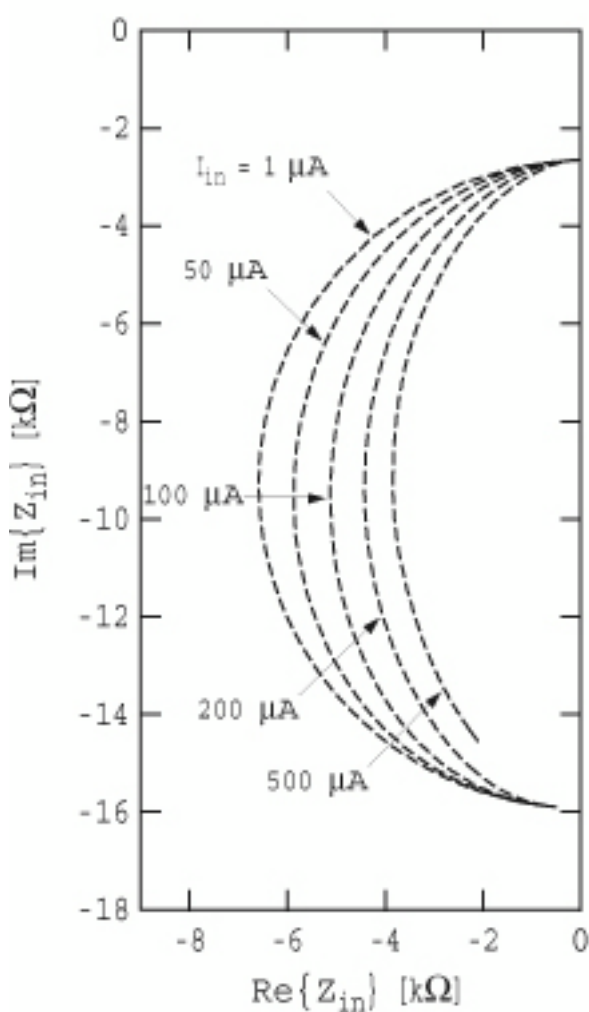
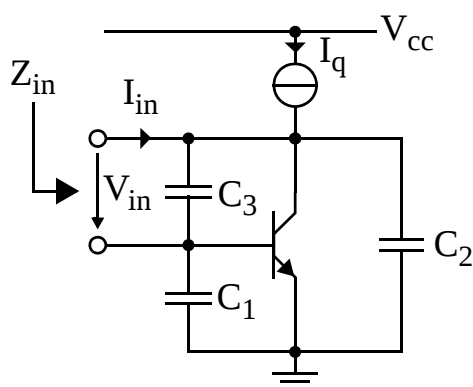
The graph in Fig. 9-9 corresponds to a linear analysis and thus does not take the non-linear effects introduced by the active element into account. These non-linear effects can be studied by simulating the circuit of Fig. 9-25 on which a sinusoidal current  $I_{in}$  is applied, and having the simulator calculate the corresponding voltage  $V_{in}$ . We can then do a FFT of the voltage  $V_{in}$  and evaluate the amplitude of the fundamental  $V_{in(1)}$ . The fundamental's impedance  $Z_{c(1)}$  is then simply given by the ratio  $V_{in(1)}/I_{in}$ . We can repeat this operation for different amplitudes  $I_{in}$  and for different polarization currents  $I_q$ . We then find the graphs of Fig. 9-26.

Fig. 9-26 a) presents the impedance points for the fundamentals  $Z_{c(1)}$  as functions of the polarization current  $I_q$  varying from  $1\ \mu\text{A}$  to  $1.728\ \text{mA}$  and for different amplitudes of the sinusoidal current signal  $I_{in}$  of  $1\ \mu\text{A}$ ,  $50\ \mu\text{A}$ ,  $100\ \mu\text{A}$ ,  $200\ \mu\text{A}$ , and  $500\ \mu\text{A}$ . We remark that for small amplitudes ( $I_{in} = 1\ \mu\text{A}$ ) we once again find the circle obtained during the linear analysis. This circle then turns into an ellipse when the amplitude of the sinusoidal signal increases due to non-linear effects.

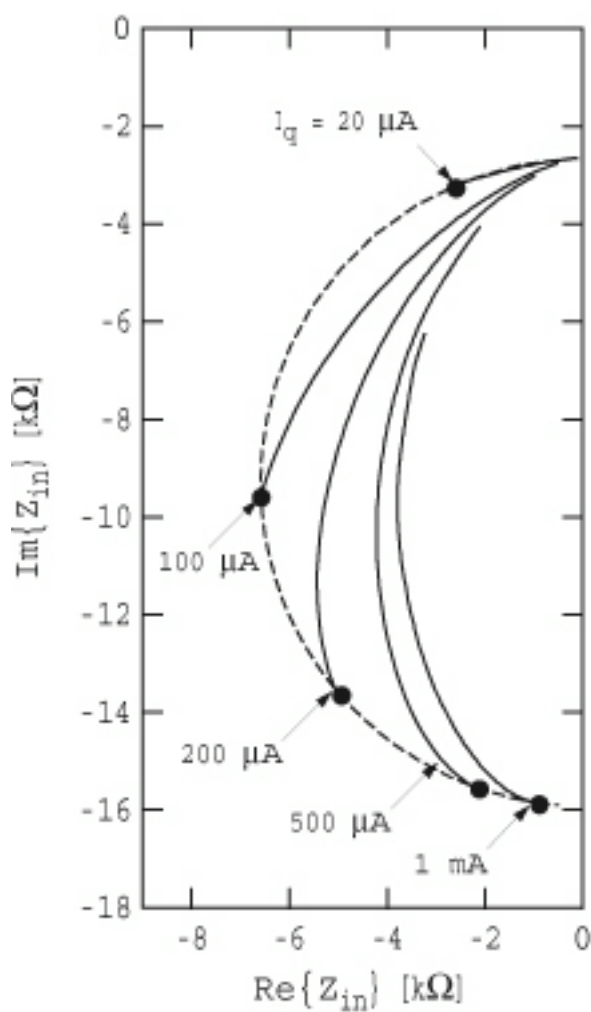
## NON-LINEAR EFFECTS (2/3)

Fig. 9-26 b) presents the location of the impedance points for the fundamentals  $Z_{c(1)}$  as a function of the amplitude of the sinusoidal current signal  $I_{in}$  varying from  $1\ \mu\text{A}$  to  $1\ \text{mA}$  for different polarization currents  $I_q$   $20\ \mu\text{A}$ ,  $100\ \mu\text{A}$ ,  $200\ \mu\text{A}$ ,  $500\ \mu\text{A}$ , and  $1\ \text{mA}$ . For each of these polarization currents, the location starts on the circle with a direction tangent to the circle when the amplitude is small, and then gets farther away from this circle. By considering for example the curve corresponding to  $I_q = 200\ \mu\text{A}$ , we remark that the operating point can be rather far from the one that we would obtain using linear analysis. It is even possible that this intersection point doesn't exist anymore due to these non-linear effects. In fact, if we assume that the resistance  $r$  is equal to  $5\ \text{k}\Omega$ , when the polarization current is greater than  $200\ \mu\text{A}$ , the trajectory no longer crosses the straight line corresponding to the impedance of the inductor. There are then no longer any operating points and the oscillations don't begin even though the polarization current is greater than the critical current.

### NON-LINEAR EFFECTS (3/3)



a)  $Z_{in(1)}$  as a function of the polarization current  $I_q$  from  $1 \mu\text{A}$  to  $1.728 \text{ mA}$  and for different amplitudes  $I_{in}$  of the applied sinusoidal signal.



b)  $Z_{in(1)}$  as a function of the amplitude  $I_{in}$  of the applied sinusoidal signal from  $1 \mu\text{A}$  to  $1 \text{ mA}$  and for different polarization currents  $I_q$ .

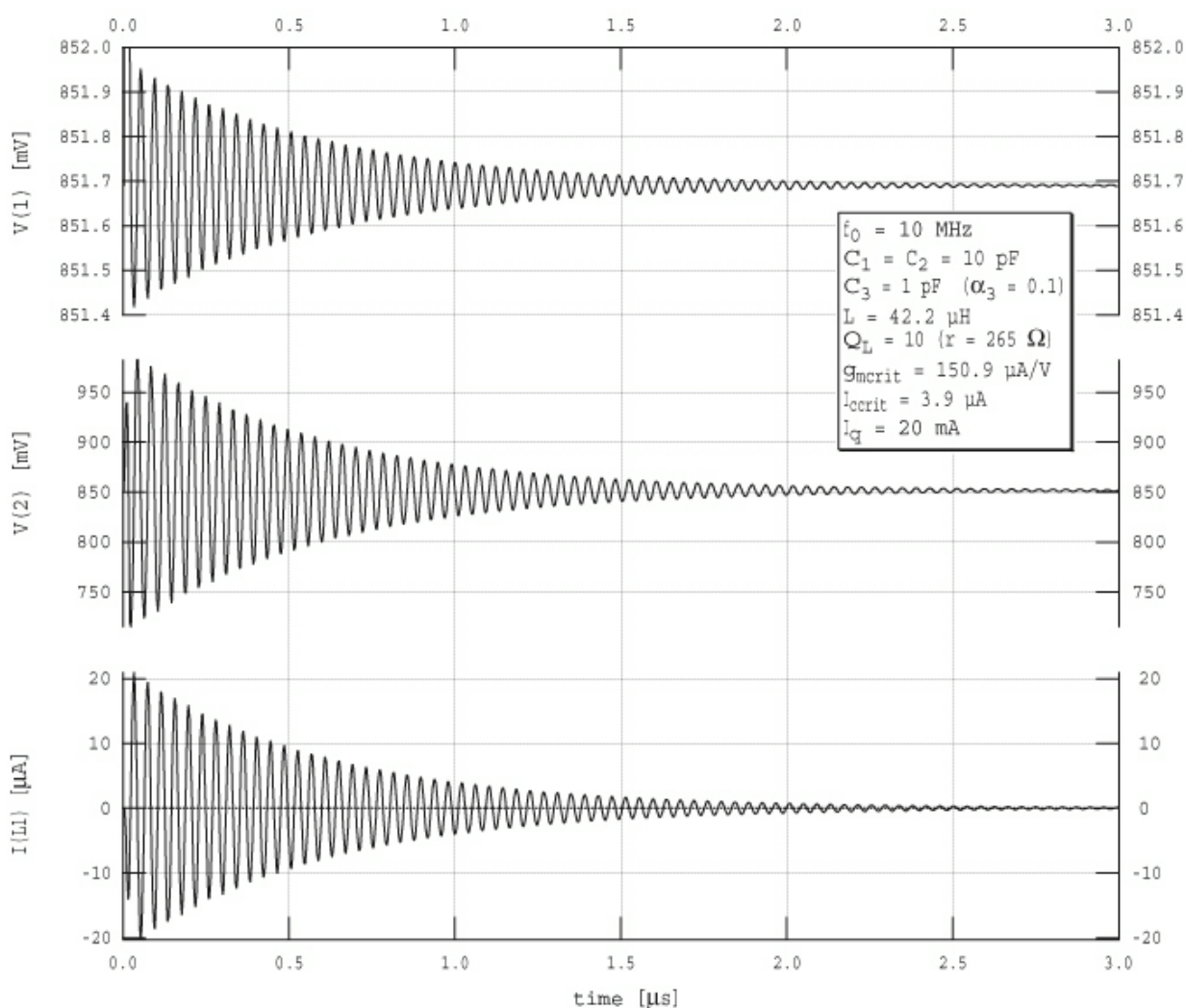
**Fig 9-26:** Impedance of the fundamental  $Z_{in(1)}$  as a function of the amplitude and the polarization current.

## EXAMPLE

It is interesting to verify with simulations that if the polarization current is large, the oscillations are not likely to begin. By taking once again the graph in Fig. 9-9, we can calculate the  $g_m$  corresponding to the intersection point B

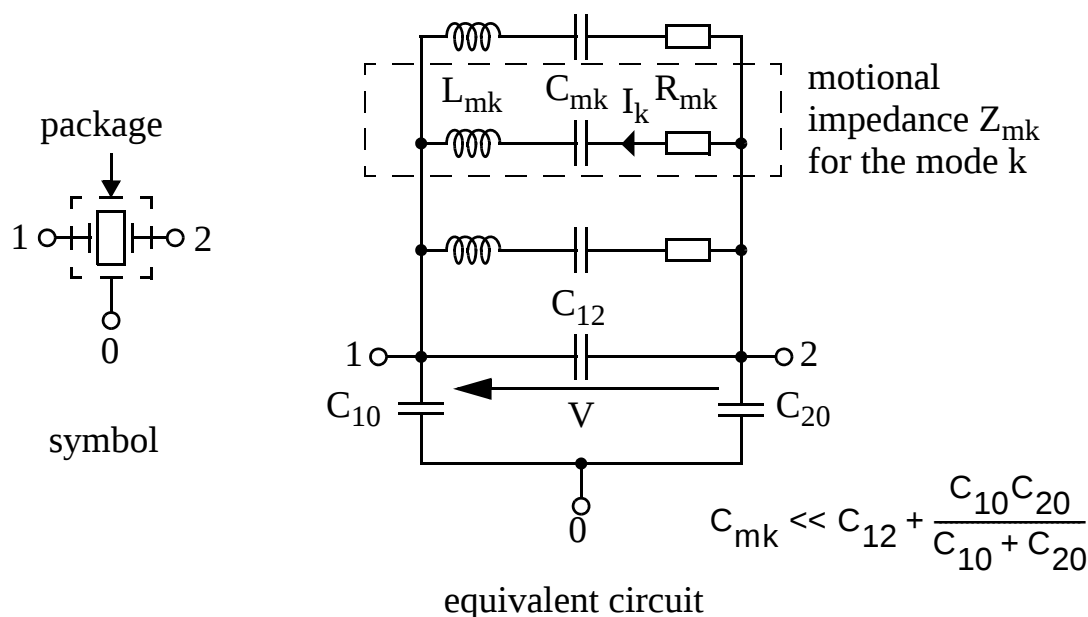
$$g_{m-unstable} \cong \frac{1}{r} \cdot \frac{\alpha_1}{2\alpha_3^2} \left[ 1 + \sqrt{1 - \left( \frac{2\alpha_3(\alpha_1 + 1)}{\alpha_1 Q_L} \right)^2} \right] \quad (9.62)$$

which in the preceding example was  $375\text{mA/V}$ , which corresponds to a polarization current of  $9.75\text{mA}$ . Fig. 9-27 corresponds to a simulation for a current of  $20\text{mA}$ . It indeed shows that the oscillations do not begin.



**Fig 9-27:** Condition  $I_q \gg I_{ccrit}$ .

## QUARTZ (1/2)



**Fig 9-28:** Symbol and equivalent circuit diagram for quartz.

The symbol for quartz and its equivalent circuit diagram are shown in Fig. 9-28. For each mechanical oscillation mode  $k$  there is a corresponding series resonant circuit  $R_k$ ,  $L_k$ ,  $C_k$  for which the series impedance is called motional impedance  $Z_{mk}$ . The inductance  $L_k$  translates the effect of the mass, the capacity  $C_k$  that of the compliance (elasticity), and the resistance  $R_k$  that of the losses of the mechanical resonator. Each mechanical oscillation mode can be characterized by its resonant frequency  $\omega_{mk}$  and its quality factor:

$$\omega_{mk} \equiv 1/\sqrt{L_k C_k} \quad Q_k \equiv \frac{\omega_{mk} L_k}{R_k} = \frac{1}{\omega_{mk} R_k C_k} \gg 1 \quad (9.63)$$

Due to the boundary effects, these frequencies  $\omega_{mk}$  are usually not whole multiples of each other. Since  $Q_k \gg 1$ , when the crystal oscillates in a certain mode  $k$ , all the other branches can be ignored, even if the voltage  $V(t)$  contains significant harmonic components. The motional current  $I_k$  can then be considered to be sinusoidal, even if the voltage  $V(t)$  is strongly distorted.

## QUARTZ (2/2)

In order to simplify the notation, we consider a single mode  $k$  and omit the corresponding index. The motional impedance  $Z_m$  is given by:

$$Z_m = R + \frac{j}{\omega_m C} \left( \frac{\omega}{\omega_m} - \frac{\omega_m}{\omega} \right) \quad (9.64)$$

We define the relative frequency spread (pulling) by:

$$p \equiv \frac{\omega - \omega_m}{\omega_m} = \frac{\Delta\omega}{\omega_m} \quad (9.65)$$

In an oscillator, the resonant frequency in radians  $\omega$  is always very close to the motional frequency  $\omega_m$  and so  $p \ll 1$ . The motional impedance is thus approximated by:

$$Z_m = R + j \frac{2p}{\omega C} \quad (9.66)$$

The mechanical oscillation energy is given by:

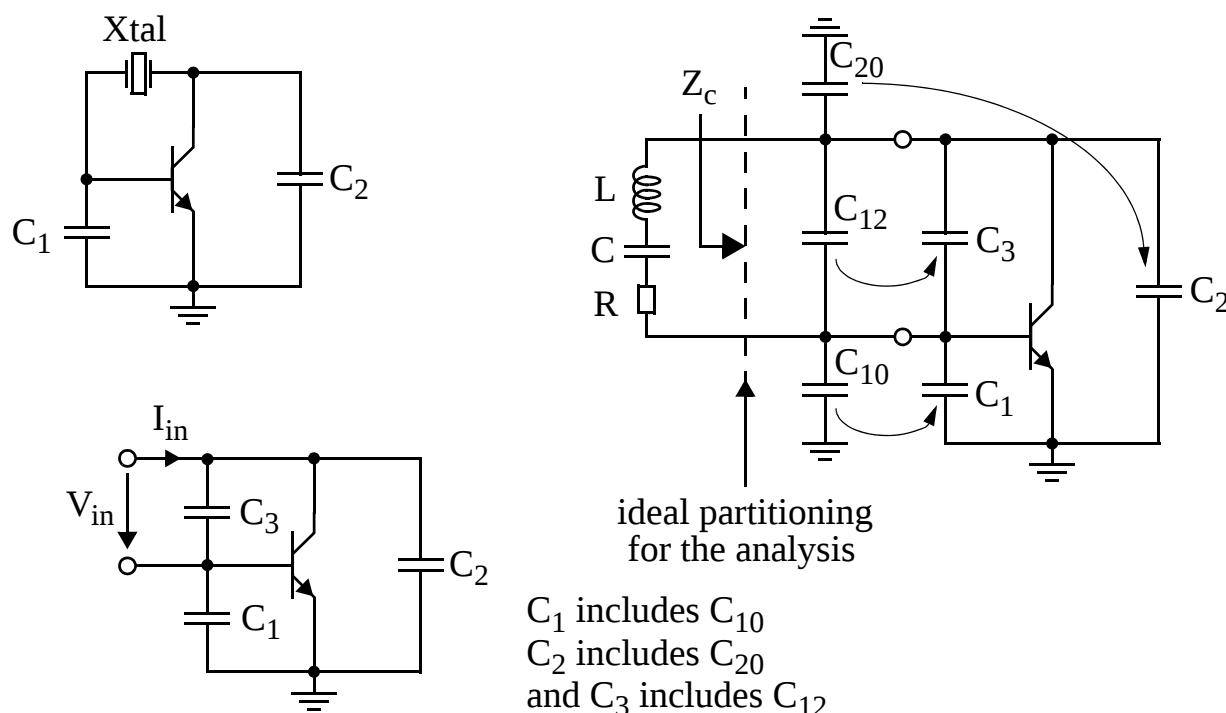
$$E_m = \frac{L|I|^2}{2} = \frac{|I|^2}{2\omega_m^2 C} \quad (9.67)$$

while the dissipated mechanical power is:

$$P_m = \frac{R|I|^2}{2} = \frac{|I|^2}{2\omega_m Q C} \quad (9.68)$$

where  $|I|$  corresponds to the amplitude of the sinusoidal current.

## THE QUARTZ OSCILLATOR

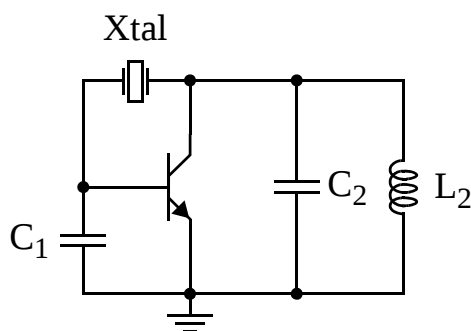


**Fig 9-29:** Quartz Pierce oscillator.

Fig. 9-29 shows the Pierce oscillator of Fig. 9-4 where the inductor has been replaced by quartz. The best way to partition the circuit in order to carry out the negative resistance method of analysis is to consider the parasitic capacitances  $C_{10}$ ,  $C_{20}$  and  $C_{12}$  to be a part of the circuit, and to isolate the series branch of the quartz resonator. Since the current flowing in this series branch can be considered to always be sinusoidal, the exchange of energy between the circuit and the series branch of the resonator always happens at the fundamental frequency. The effects of the non-linearities of the active circuit can thus be entirely characterized by the impedance  $Z_{c(1)} \equiv V_{in(1)}/I_{in}$  where  $V_{in(1)}$  is the complex amplitude of the fundamental. In addition, the frequency dependence of  $Z_{c(1)}$  around  $\omega_m$  is orders of magnitude smaller than that of  $Z_m$  and so the frequency  $\omega$  can be considered to be constant and equal to  $\omega_m$  during the evaluation of  $Z_{c(1)}$ , while the frequency dependence of  $Z_m$  is expressed in terms of the "pulling"  $p$  according to (9.65).



## “OVERTONE” QUARTZ OSCILLATOR



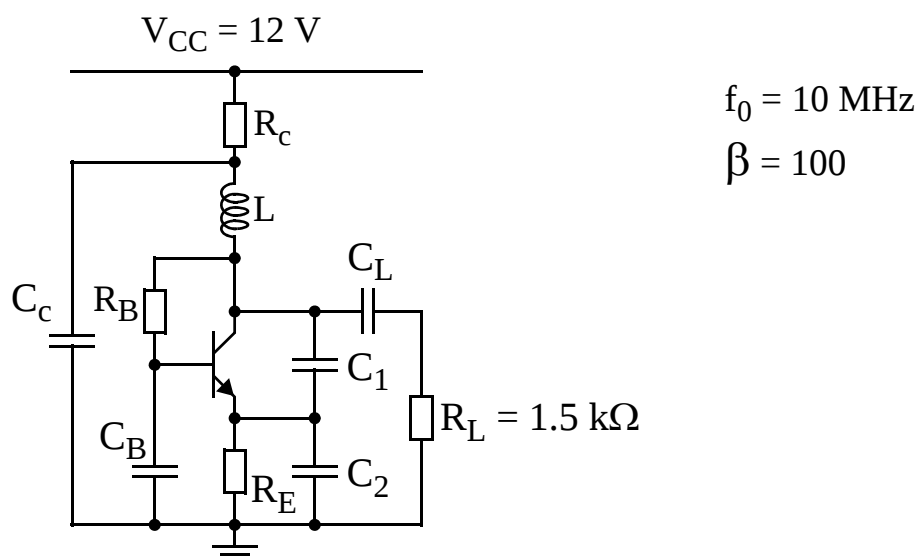
**Fig 9-31:** “Overtone” Pierce oscillator.

The resonant frequency of quartz oscillators for the fundamental mode is typically limited to 100 MHz. Nevertheless, we can force the resonator to oscillate not at the fundamental, but at a higher mode (generally the 3<sup>rd</sup> or 5<sup>th</sup>). The oscillation condition (9.47) requires that  $X_1$  and  $X_2$  have the same sign, and that  $X_1 + X_2 + X_3 = 0$ . For the resonator to oscillate at a higher mode, it is necessary for the parallel LC circuit to have a negative reactance (or be capacitive) at the overtone frequency. We can evaluate the oscillation frequency by replacing  $X_2$  and  $X_3$  by

$$X_2 = \frac{\omega L_2}{1 - (\omega/\omega_{LC})^2} \quad X_3 = \omega L_m \cdot \left[ 1 - \left( \frac{\omega_m}{\omega} \right)^2 \right] \quad (9.69)$$

where  $\omega_{LC} \equiv 1/\sqrt{L_2 C_2}$  and where we have ignored the capacitances  $C_{12}$  and  $C_3$ . So that  $X_2 < 0$  at the oscillation frequency, it is necessary that  $\omega_{LC} < \omega_m$ .  $X_3$  is thus positive when  $\omega > \omega_m$ . The oscillation frequency will then be slightly greater than  $\omega_m$  and also greater than  $\omega_{LC}$ . Of course this analysis is a rough approximation, because it assumes that the transistor has been reduced to a simple transconductance, which is definitely not the case at overtone frequencies.

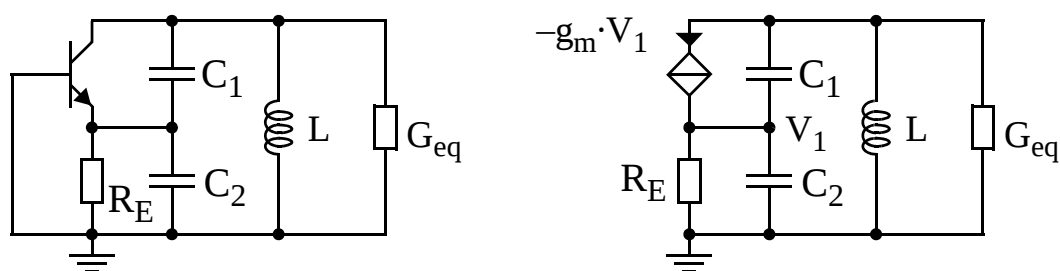
## EXAMPLE: COMMON BASE OSCILLATOR



**Fig 9-32:** *Common base oscillator.*

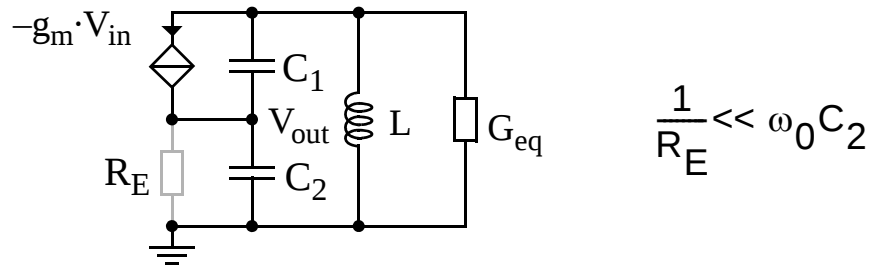
We would like to analyze and dimension the oscillator in Fig. 9-32 for a frequency of 10 MHz. We will compare the results obtained using the three proposed methods of analysis.

The first step of the linear analysis is to draw the small-signal diagram. We then get the diagram in Fig. 9-33.



**Fig 9-33:** *Equivalent small-signal diagram.*

## ANALYSIS USING THE BARKHAUSEN CRITERION



**Fig 9-34:** *Small-signal equivalent circuit diagram in open loop.*

In the diagram in Fig. 9-33, we can virtually open the loop between the voltage  $V_1$  and the current controlled by this voltage. We then get the diagram presented in Fig. 9-34. We can simplify the calculations by assuming that  $1/R_E \ll \omega_0 C_2$  which is the same as neglecting  $R_E$ . The open-loop gain is thus given by:

$$GH = -g_m \frac{1 + sG_{eq}L}{s(C_1 C_2 L s^2 + G_{eq}L(C_1 + C_2)s + C_1 + C_2)} \quad (9.70)$$

By applying the Barkhausen criterion we find the conditions for the real and imaginary parts:

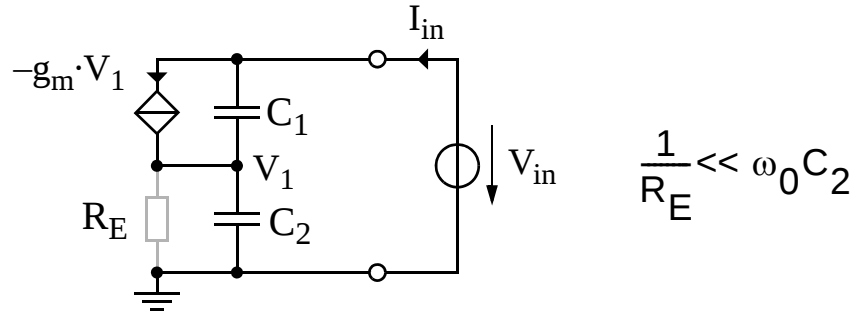
$$\begin{cases} C_1 C_2 L \omega^2 - (g_m G_{eq} L + C_1 + C_2) = 0 \\ G_{eq} L (C_1 + C_2) \omega^2 - g_m = 0 \end{cases} \quad (9.71)$$

We can solve (9.71) for  $g_m$  and  $\omega$  and find  $\omega_0$  and  $g_{mcrit}$ :

$$\begin{aligned} \omega_0 &= \frac{1}{\sqrt{L(C_{12} - G_{eq}^2 L)}} \\ g_{mcrit} &= \frac{G_{eq}(C_1 + C_2)}{C_{12} - G_{eq}^2 L} = \frac{\omega_0(C_1 + C_2)}{Q} \end{aligned} \quad (9.72)$$

where  $C_{12} \equiv C_1 C_2 / (C_1 + C_2)$  and  $Q \equiv 1 / (\omega_0 G_{eq} L)$ . We remark that a solution exists only for  $C_{12} > G_{eq}^2 L$ .

## THE NEGATIVE CONDUCTANCE METHOD (1/2)



**Fig 9-35:** Calculation of the parallel admittance.

The losses of the parallel resonant circuit are represented in the diagram in Fig. 9-33 by the conductance  $G_{eq}$ . These losses must be compensated by a negative conductance provided by the circuit in Fig. 9-35. By assuming once again that  $1/R_E \ll \omega_0 C_2$  and that  $g_m R_E \gg 1$  we find the input admittance  $Y_{in}$ :

$$Y_{in} \equiv G_{in} + jB_{in} \cong \frac{-\omega^2 C_1 C_2}{g_m + j\omega(C_1 + C_2)} \quad (9.73)$$

The (negative) conductance  $G_{in}$  must thus be equal to  $-G_{eq}$  and the susceptance  $B_{in}$  must be equal to  $\frac{1}{\omega L}$ :

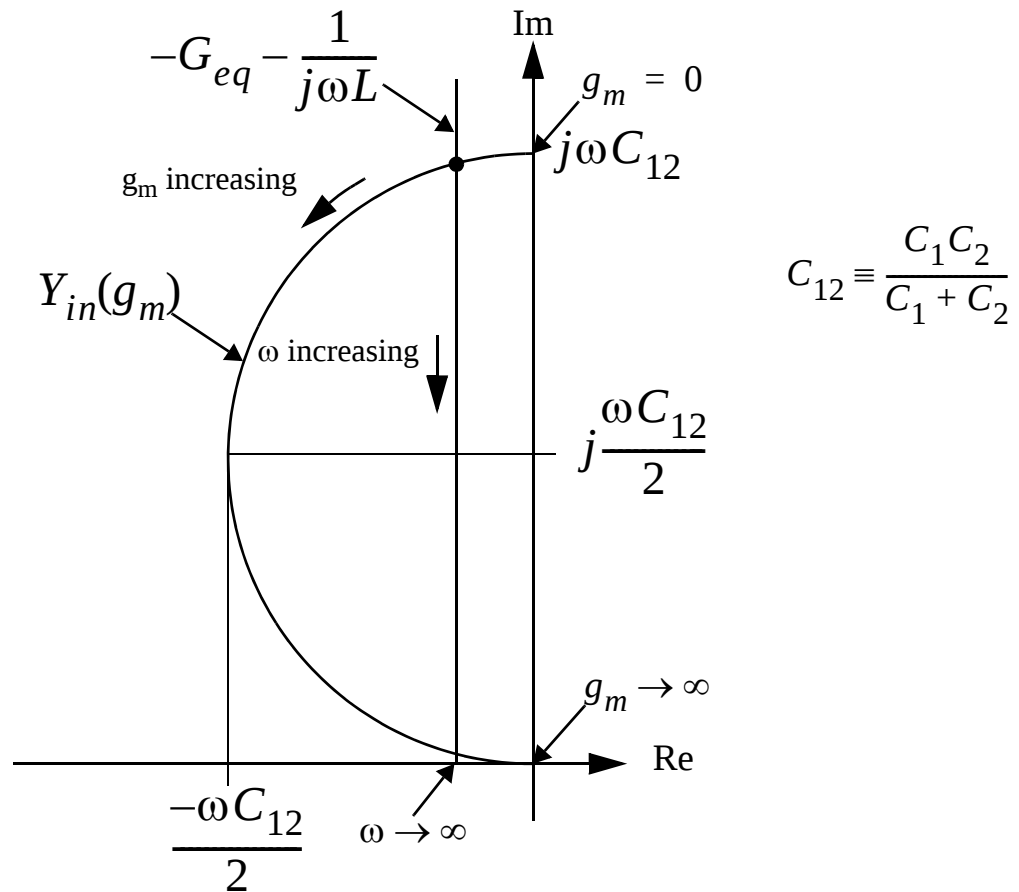
$$\begin{cases} G_{in} \cong \frac{-\omega^2 C_1 C_2 g_m}{g_m^2 + \omega^2 (C_1 + C_2)^2} = -G_{eq} \\ B_{in} \cong \frac{\omega^3 C_1 C_2 (C_1 + C_2)}{g_m^2 + \omega^2 (C_1 + C_2)^2} = \frac{1}{\omega L} \end{cases} \quad (9.74)$$

from which we find the resonant frequency in radians, and the critical transconductance. We find the same results given by (9.72) and obtained by using the Barkhausen criterion.

## THE NEGATIVE CONDUCTANCE METHOD (2/2)

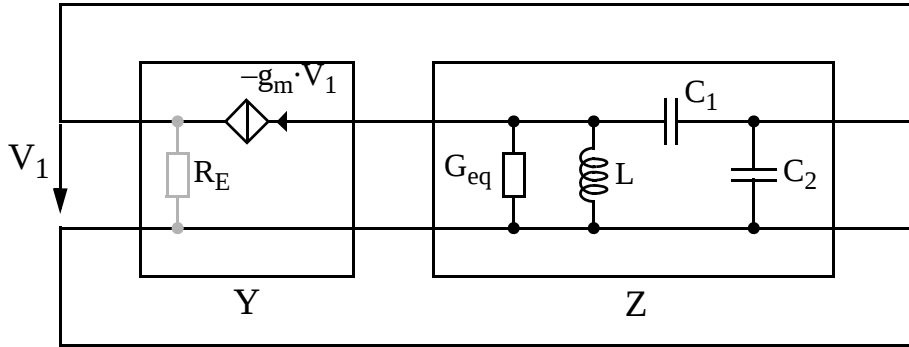
As for the Pierce oscillator, we can represent the admittance points  $Y_{in}$  in the complex plane as a function of the parameter  $g_m$  (cf Fig. 9-36). In the same plane, we can also plot the admittance points  $-G_{eq} - 1/j\omega L$  as a function of  $\omega$ . The intersection presented in Fig. 9-36 then corresponds to the operating point (for small signals). From this figure, we can also find an existence condition for this operating point. In fact, the parallel conductance  $G_{eq}$  must remain smaller than  $-\omega C_{12}/2$ . We also remark that the approximation of the circle by the tangent at  $g_m = 0$  gives a value close to  $\omega_0$ , that is to say:

$$\omega_0 \cong 1/\sqrt{LC_{12}} \quad (9.75)$$



**Fig 9-36:**  $Y_{in}$  points as a function of  $g_m$  and  $-Y_L = -G_{eq} - 1/(j\omega L)$  as a function of  $\omega$ .

## ANALYSIS BY THE Y AND Z PARAMETER METHOD



**Fig 9-37:** Calculation of the parallel admittance.

The diagram in Fig. 9-33 is redrawn in Fig. 9-37 to highlight the elements of the Y and Z matrices. By ignoring the resistance  $R_E$ , the Y and Z matrices are given by:

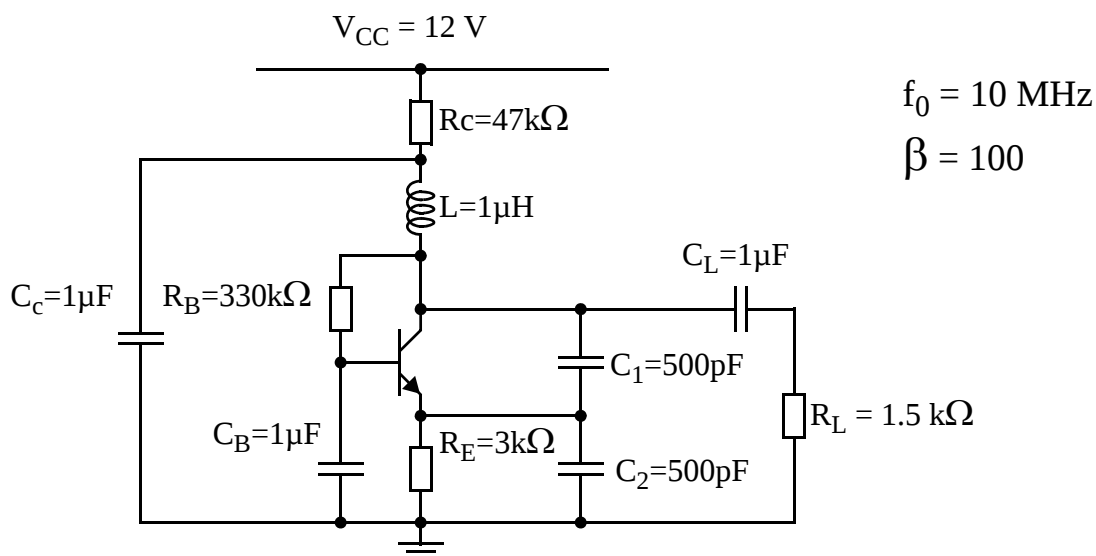
$$Y = \begin{bmatrix} g_m & 0 \\ -g_m & 0 \end{bmatrix} \quad Z = \begin{bmatrix} \frac{j\omega L(C_1 + C_2)}{N(\omega)} & \frac{j\omega LC_1}{N(\omega)} \\ \frac{j\omega LC_1}{N(\omega)} & \frac{\omega G_{eq}L - j(1 - \omega^2 LC_1)}{\omega N(\omega)} \end{bmatrix} \quad (9.76)$$

where  $N(\omega) \equiv (C_1 + C_2) - \omega^2 C_1 C_2 L + j\omega G_{eq}L(C_1 + C_2)$ . By applying the relation (9.39) and separating the real and imaginary parts, we find the following conditions:

$$\begin{cases} -g_m + \omega^2 G_{eq}L(C_1 + C_2) = 0 \\ -g_m G_{eq}L + \omega^2 LC_1 C_2 - C_1 - C_2 = 0 \end{cases} \quad (9.77)$$

from which we can find the resonant frequency and the critical transconductance. Once again, we get the same results as with the analysis using the Barkhausen criterion, given by (9.72).

## DIMENSIONING OF THE COMMON BASE OSCILLATOR



**Fig 9-38:** Dimensioned common base oscillator.

We simplify the calculations by assuming that  $\omega_0 \cong 1/\sqrt{LC_{12}}$ . We remark from (9.72) that, as for the Pierce oscillator, the transconductance is minimum when  $C_1 = C_2$  is minimum. For  $C_1 = C_2 = 500 \text{ pF}$  we find  $L = 1 \mu\text{H}$ . To satisfy the condition  $1/R_E \ll \omega_0 C_2$  we will take  $R_E = 3 \text{ k}\Omega$ . In addition,

$$G_{eq} = 1/R_b + 1/R_L \cong 1/R_L = 0.667 \mu\text{A/V}$$

The critical transconductance is approximately given by:

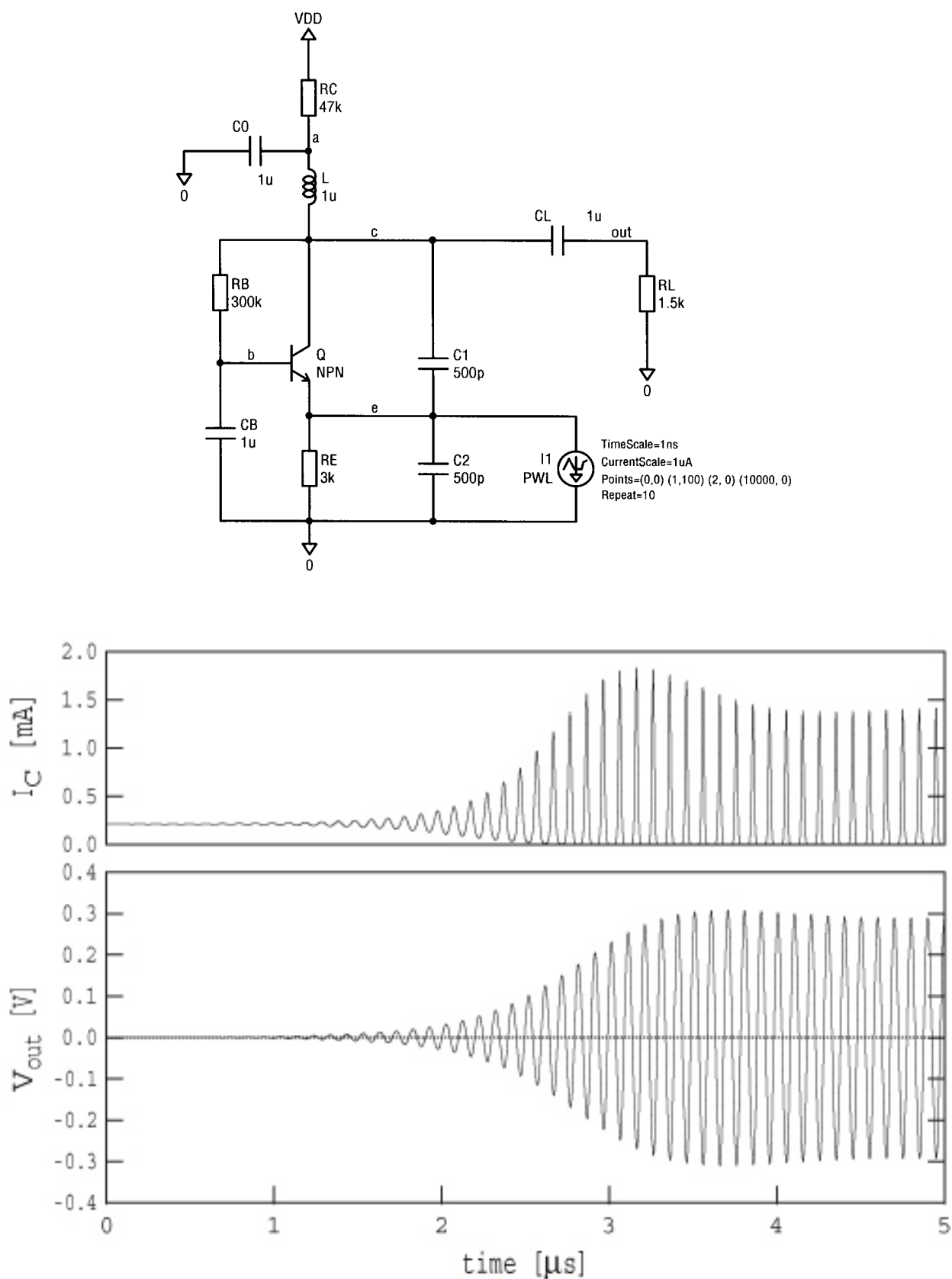
$$g_{mcrit} \cong G_{eq}(C_1 + C_2)/C_{12} = 4G_{eq} = 4/R_L$$

To start up the oscillations we take

$$g_m = 3 \cdot g_{mcrit} = 12/R_L = 8 \text{ mA/V}$$

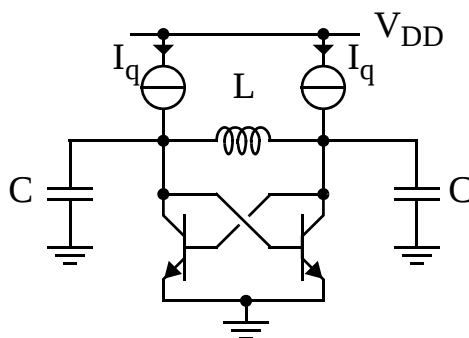
from which we find the current  $I_{Cq} = 208 \mu\text{A}$ . The base current being  $\beta = 100$  times smaller, we take  $R_B = \beta R_E = 300 \text{ k}\Omega$ . With  $U_j = 0.6 \text{ V}$  we find  $R_C = 47 \text{ k}\Omega$ . Simulation results are given in Fig. 9-39. The average period over 10 periods is  $993 \text{ ns}$  corresponding to  $10.07 \text{ MHz}$  while the amplitude is about  $300 \text{ mV}$ .

# SIMULATION OF THE COMMON BASE OSCILLATOR



**Fig 9-39:** *Simulation of the oscillator*

## LC OSCILLATOR WITH TWO TRANSISTORS



**Fig 9-40:** *LC oscillator with two transistors.*

When we allow ourselves to use more than one transistor for the realization of an LC oscillator, we can go back to the diagram presented in Fig. 9-40. This diagram is very much used for realizing integrated VCOs oscillating around 1 GHz. This oscillator is interesting because it consists of only two active nodes (in contrast with the "three point" oscillator which has 3). All the parasitic capacitances of these two nodes can thus be absorbed in the functional capacitances. By assuming that there is no load, only the losses due to the series resistance  $r$  of the inductor must be compensated. We can thus transform the series resistance to a parallel resistance  $R_p = (\omega L)^2 / r$  or  $R_p = Q_L \omega L$ . The corresponding conductance will need to be compensated by the negative conductance of the circuit which has the value  $-g_m/2$ . The resonant frequency in radians is simply given by:

$$\omega_0 = \sqrt{\frac{2}{LC}} \quad (9.78)$$

and the critical transconductance by:

$$g_{mcrit} = \frac{\omega_0 C}{Q_L} \quad (9.79)$$

## IMPORTANCE OF PHASE NOISE IN TRANSMITTERS

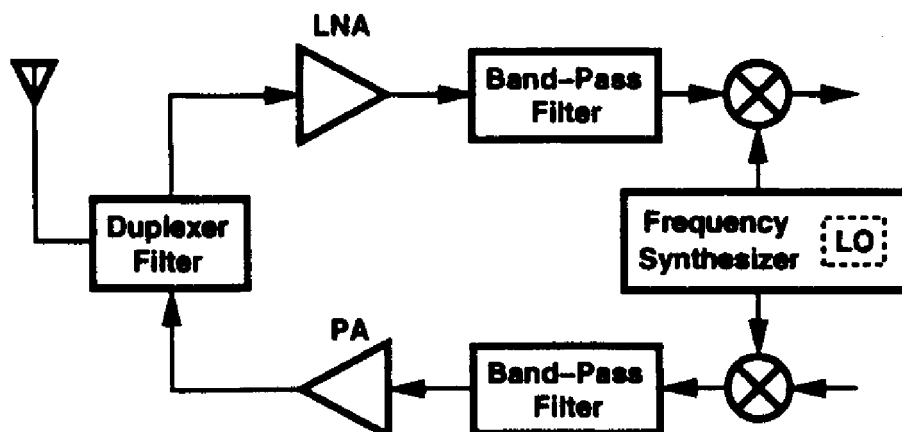


Fig 9-41: General diagram of a transmitter.

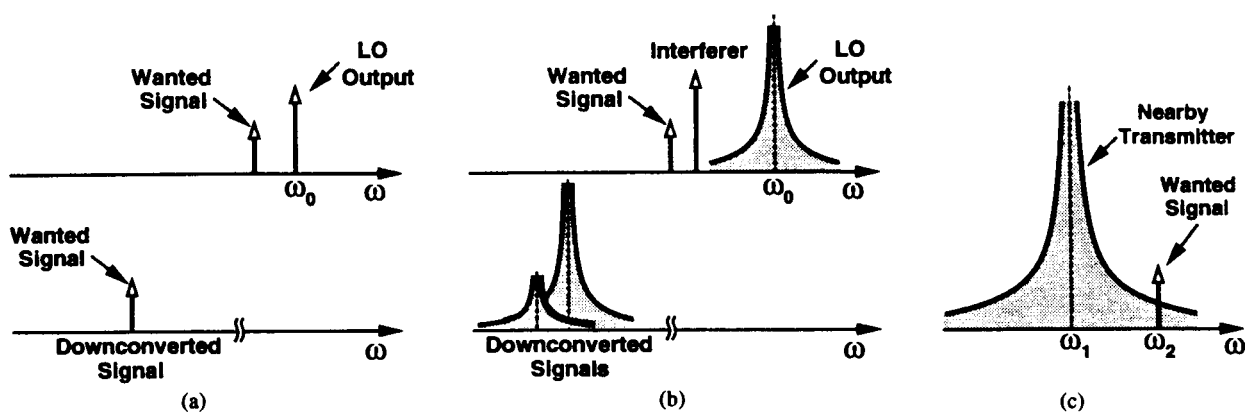
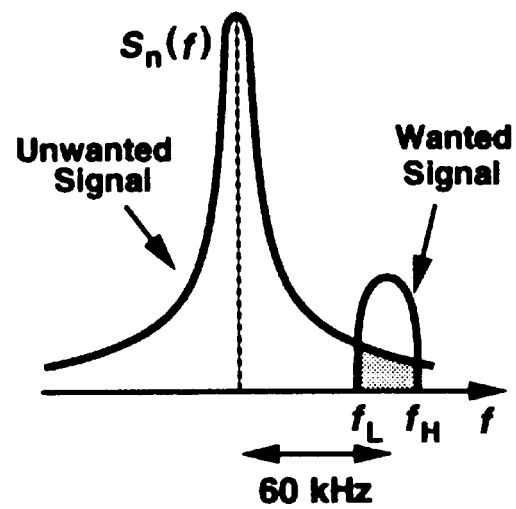


Fig 9-42: a) Frequency conversion by an ideal oscillator.  
 b) Reciprocal mixing.  
 c) Effect of phase noise in transmitters.

## SIGNAL-TO-NOISE RATIO



**Fig 9-43:** *Degradation of the SNR by the phase noise.*

## EXAMPLE OF PHASE NOISE

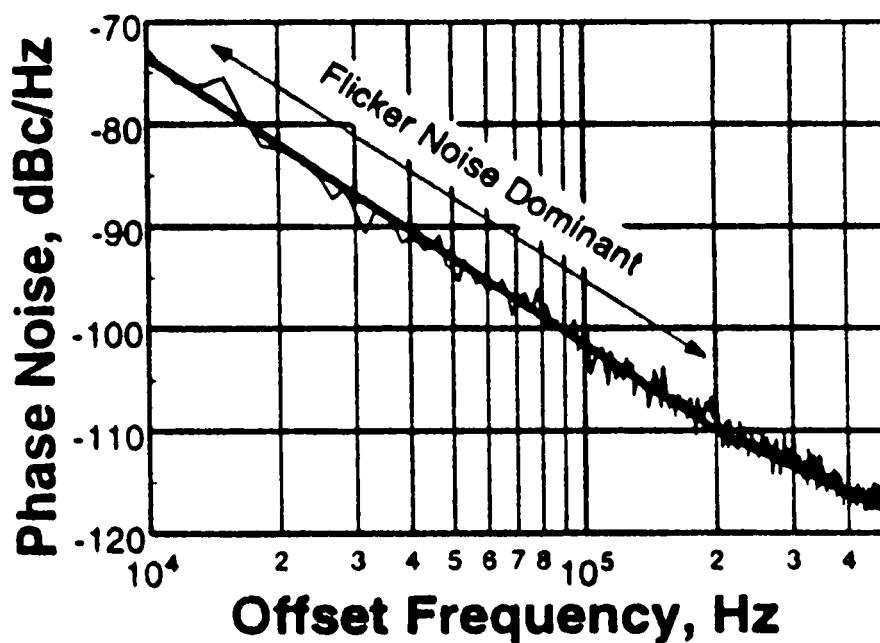


Fig 9-44: Phase noise measured in an LC oscillator.

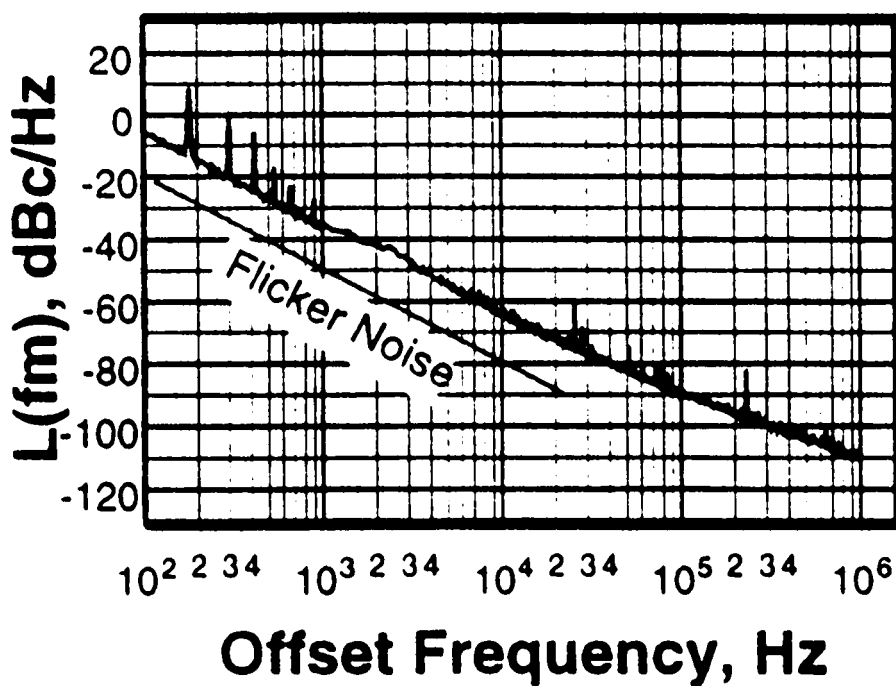


Fig 9-45: Phase noise measured in a ring oscillator.